Multiview Triangulation with Uncertain Data

Liwei Zhang[1,2,3], Jianhua Zhang[4], Bo Chen[5], Zhenli Lu[6], Ying Hu[1,2], Jianwei Zhang[3]

***Abstract*— The traditional triangulation algorithms in multiview geometry problems have the drawback that its solution is locally optimal. Robust Optimization is a specific and relatively novel methodology for handling optimization problems with uncertain data. The key idea of robust optimization is to find the best possible performance in the worst case. In this paper, we propose a novel approach which solves the triangulation problems with perturbational data employing robust optimization. The main advantage of this method is global optimality under the perturbational data. Good performance has been demonstrated by experimental results for synthetic and real data, respectively.**

***Index Terms*— robust optimization, multiview geometry, global optimality, triangulation, outlier removal.**

I. Introduction

Computer vision aims to bestow perceptive ability on computer. Some remarkable progress has been achieved in the area of multiview geometry in 1990's. But the traditional methods have two drawbacks: (1)only local solution can be achieved when we employ some traditional methods like least square method; (2) the algorithm designed for certain data, which means no perturbation.

There are two reasons responsible for the inaccurate solution: one is the robustness of the algorithm, i.e., whether the algorithm is robust to data; the other is the data are uncertain in itself. The most common reasons for data uncertainty are as follows:

- Measurement error. Some of data entries can not be measured or sensed by the sensor when the problem is solved and hence are replaced with their forecasts. These data entries are thus subject to measured errors;
- Truncation error. Most numerical processes are not accurate. There are truncation errors in the numerical process since the computer can not present some variables accurately.
- Propagation of the error. The errors can be propagated or enhanced by the imperfect algorithm.

Robust Optimization is a specific and relatively novel methodology for handling optimization problems with uncertain data. The key idea of robust optimization is to find the best possible performance in the worst case. In convex optimization, several 'generic' families of conic problems are of special interest, both from the viewpoint of theory and applications. Moreover, in many cases the 'algorithmic duality machinery' allows to understand more deeply the original model, to convert it into equivalent forms better suited for numerical processing, etc. The relative simplicity of the underlying cones also enables one to develop efficient computational methods for the corresponding conic problems. The Second Order Cone Programming (SOCP) will be employed in this paper since it is the most useful form in computer vision communities.

Typically, the data in multiview geometry problems is not known exactly. The uncertainty of data in multiview geometry problems is common, like the truncation of the pixels value, the process of point correspondence in stereo vision system, etc. This means we can not obtain accurate solution if the algorithm is robustless to perturbational data. The aim of this paper is to investigate the use of robust optimization techniques in triangulation problems. One key advantage of robust optimization methods is the existence of a global optimal solution. Hence, there is no problem with falling into a local minimum unlike traditional algorithms. Robust optimization also has the advantage that the algorithm has tolerance for uncertain data, which means robustness and adaptability. On the computational side of things, robust methods are relatively fast and quite easy to implement, since there is software like SeDuMi [1], [2] or CVX [3]. In this paper, we present the use of robust optimization in the core multiview geometry problems: triangulation.

The rest of this paper will be organized as follows: section 2 discusses the related work, section 3 presents some notation and concepts of robust optimization. In section 4, the formula of the triangulation problem by employing robust optimization are introduced. Virtual and real data experiments are conducted in section 5 to evaluate and compare the efficiency and precision of the proposed approach and, finally, in section 6, conclusions and future work are presented.

II. Related Work

The triangulation problem is the basic and core problem in computer community, which refers to determine a 3D space point given its projections onto two or more images. In theory, the triangulation problem is trivial. Each point in an image corresponds to a line in 3D space such that all points on the line are projected on that first point in the image. If a pair of corresponding points in two or more images can be found, they must be the projection of a common 3D point.

[1,2] Liwei Zhang and Ying Hu are with Shenzhen Institute of Advanced Integration Technology, Chinese Academy of Sciences, Shenzhen, China and The Chinese University of Hong Kong, Hong Kong, China {lw.zhang, ying.hu}@siat.ac.cn

[3] Jianwei Zhang is with TAMS, Department of Informatics, University of Hamburg, Hamburg, Germany zhang@informatik.uni-hamburg.de

[4] Jianhua Zhang is with Zhejiang University of Technology, Hangzhou, China zjh@zjut.edu.cn

[5] Bo Chen is with College of Mathematics and Computational Science, Shenzhen University, Shenzhen, China chenbo@szu.edu.cn

[6] Zhenli Lu is with Institute of Electronics and Telematics Engineering of Aveiro, University of Aveiro, Portugal zhenli.lu@ua.pt

The set of lines generated by the image points must intersect at the 3D point and the coordinates of the 3D point can be computed. However, the coordinates of image points maybe corrupted by noisy data, i.e., they cannot be measured with arbitrary accuracy. As a consequence, the lines generated by the corresponding image points do not always intersect in 3D space.

In chapter 12 of [4], the basic idea, merits and drawbacks of triangulation in traditional vision are discussed in detail. An optimal method was given in [5], and it is shown that an optimal solution can be obtained via the root of a sixth-degree polynomial, thus avoiding a non-linear minimization of a cost function. This method was generalized to 3 views by [6]. [7] introduced a new method that the non-convex problem was approximated by convex linear matrix inequalities (LMI). But there is no guarantee of global optimality.

The drawback of the traditional methods is that the cost function is non-convex, which causes the solution to be locally optimal. In recent years, the convex optimization theory [8] has been applied to computer vision problems and useful results have been attained [9], [10], [11], [7], [12], [13], [14], [15], [16], [17].

The Branch and Bound Algorithm was used to solve the triangulation problem in traditional vision in [18]. Sim [19] gave a detailed discussion of the Multi-view Geometry and convex optimization problems under the L_∞ norm in his Ph.D dissertation. In all of these methods, a single global minimum exists and it may be found by SOCP(Second-Order Cone Programming).

The triangulation problem in omnidirectional vision is more complicated because the points in space are first reflected by a mirror. Zhang [20], [21], [22] transformed the triangulation problem in omnidirectional vision to a convex optimization problem based on the property of global optimality of SOCP. The key idea is to estimate space points from image points using SOCP under the L_∞ norm. The main advantages of this algorithm are global optimality and a wide field of view, which cannot be provided by traditional triangulation method under traditional vision system.

All the above methods are used to solve the triangulation problem with certain data. In practice, however, the data are corrupted by other issues. For instance, geometric noise from lens distortion or interest point detection error, lead to inaccuracies in the measured image coordinates. The researchers also found that the convex optimization methods under L_∞ norm are sensitive to noise data. In this work, we solve the triangulation problem with uncertain(perturbational) data using robust optimization.

III. Robust Optimization

In this section, some notations and concepts of robust optimization are presented. For more details, the reader is referred to the excellent book [23] for robust optimization problems.

A conic optimization problem is one of the form

$$\min_x \{c^T x : b - Ax \in \mathcal{K}\}$$

where $\mathcal{K}$ is convex cone. There is another most common form

$$\|Ax + b\|_2 \le c^T x + d$$

In Robust Optimization, an uncertain Conic Programming problem is defined as a collection

$$\left\{ \min_x \{c^T x : b - Ax \in \mathcal{K}\} : (c, A, b) \in \mathcal{U} \right\}$$

of Conic Programming programs of a common structure with the data (c, A, b) varying in a given uncertainty set $\mathcal{U}$.

In this paper, we focus on computational tractability of the triangulation problems under uncertain data. We will use the assumptions in the following sections:

$$\underbrace{\|A(\zeta)x + b(\zeta)\|_2}_{\equiv \alpha(x)\zeta + \beta(x)} \le \underbrace{c^T(\zeta)x + d(\zeta)}_{\equiv \sigma^T(x)\zeta + \delta(x)} \quad (1)$$

where $A(\zeta) \in \Re^{k\times n}, b(\zeta) \in \Re^k, c(\zeta) \in \Re^n, d(\zeta) \in \Re^n$ are affine in perturbation variable ζ, so that $\alpha(x), \beta(x), \sigma(x), \delta(x)$ are affine in the decision vector x.

We assume the following conditions are satisfied:

- The uncertainty is side-wise: the perturbation set $Z = Z^{left} \times Z^{right}$ is the direct product of two sets (so that the perturbation vector ζ is split into blocks $\eta \in Z^{left}$ and $\chi \in Z^{right}$), with the left hand side data $A(\zeta), b(\zeta)$ depending solely on η and the right hand side data $c(\zeta), d(\zeta)$ depending solely on χ, so that Eqn.(1) reads
$$\underbrace{\|A(\zeta)x + b(\zeta)\|_2}_{\equiv \alpha(x)\eta + \beta(x)} \le \underbrace{c^T(\zeta)x + d(\zeta)}_{\equiv \sigma^T(x)\chi + \delta(x)}$$
- The right hand side perturbation set is
$$Z^{right} = \{\chi | \exists u : P\chi + Qu + p \in \mathbf{K}\}$$
- The left hand side uncertainty is a simple interval one: $Z^{left} = \{\eta = [\delta A, \delta b] : |(\delta A)_{ij}| \le \delta_{ij}, 1 \le i \le k, 1 \le j \le n, \; |(\delta b)_i| \le \delta_i, 1 \le i \le k\}$
In other words, every entry in the left hand side data [A, b] of Eqn.(1), independently of all other entries, runs through a given segment centered at the nominal value of the entry: $[A(\zeta), b(\zeta)] = [A^n, b^n] + [\delta A, \delta b]$.

Theorem 1. Under above assumptions on the perturbation set Z, the RC of the uncertain Eqn.(1) is equivalent to the following explicit system of conic quadratic and linear constraints in variables y, z, τ, v:

$$\begin{aligned} &\tau + p^T v \le \delta(x), && P^T v = \sigma(x) \\ &Q^T v = 0, && v \in K_* \\ &z_i \ge |(A^n x + b^n)_i| + \delta_i + \sum_{j=1}^n |\delta_{ij} x|, && 1 \le i \le k \\ &\|z\|_2 \le \tau \end{aligned}$$

The proof of this theorem can be found in [23].

IV. Triangulation

In this section, triangulation problem under L_2 and L_∞ norm are first introduced. Triangulation with uncertain data is presented in the following subsection since it includes SOCP problems as a necessary step.

A. Triangulation under L_2 and L_∞ Norm

The problem of triangulation estimation can be formulated as follows:

Given image point correspondences $\vec{m_i}, i = 1, \ldots, n$, the triangulation problem is to find space point $\vec{X}$, such that $\vec{m_i} \sim P_i\vec{X}$, where symbol $\sim$ means up to a scale factor. The rationale of twoview triangulation problem is shown in Fig. 1.

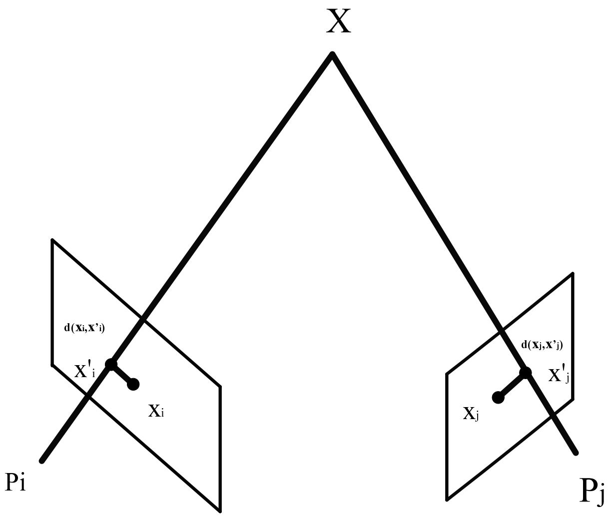

Fig. 1. Rationale of twoview triangulation problem

Let $P_i\vec{X} = (p_i^{1T}\vec{X}, p_i^{2T}\vec{X}, p_i^{3T}\vec{X})^T$. Then

$$\begin{aligned} \epsilon_i &= d(\vec{x_i}, P_i\vec{X}) \\ &= \sqrt{(x_i - \frac{p^{1T}\vec{X}}{p^{3T}\vec{X}})^2 + (y_i - \frac{p^{2T}\vec{X}}{p^{3T}\vec{X}})^2} \\ &= \sqrt{\frac{(p^{1T}\vec{X} - x_i p^{3T}\vec{X})^2 + (p^{2T}\vec{X} - y_i p^{3T}\vec{X})^2}{(p^{3T}\vec{X})^2}} \\ &= \sqrt{\frac{f_{i1}^2(x) + f_{i2}^2(x)}{f_{i3}^2(x)}} \end{aligned} \quad (2)$$

The triangulation problem now is to minimize the above cost function to find the solution. The summation of the squared error under the L_2 norm is

$$\|\epsilon\|_2^2 = \sum_{i=1}^{m} \epsilon_i^2 = \sum_{i=1}^{m} d(\vec{x_i}, P_i\vec{X})^2$$

This type of cost function has multiple local minima as is shown in [4]. We can use the L_∞ norm to avoid local minima. The cost function under the L_∞ norm is

$$\|\epsilon\|_\infty = \max_i |\epsilon_i| = \max_i |d(\vec{x_i}, P_i\vec{X})|$$

or, equivalently,

$$f_0(x) = \max_{i=1,\ldots,m} d(\vec{x_i}, P_i\vec{X})^2 = \max_{i=1,\ldots,m} \frac{f_{i1}^2(x) + f_{i2}^2(x)}{f_{i3}^2(x)} \quad (3)$$

where $x = (x_1, x_2, x_3)^T$ is the first three variables of $\vec{X}$. We find that the function $f_{i1}(x), f_{i2}(x), f_{i3}(x)$ is the affine function of x. In computer vision communities, the world points must be in front of all cameras $P_i, i = 1, \ldots, n$, this constraint is the *cheirality* constraint [24]. In the triangulation problem, the *cheirality* constraint requires that either $f_{i3}(x) > 0$ or $f_{i3}(x) < 0$ for all i.

The final optimization problem is

$$\begin{aligned} \min_x \quad & \max_{i=1,\ldots,n} f_i(x) \\ s.t. \quad & f_{i3}(x) > 0 \end{aligned}$$

According to [17], the cost function $f_0(x) = \max_i f_i(x)$ is a quasi-convex function over the convex domain $\{x | f_{i3}(x) > 0, \forall i = 1, \ldots, n\}$. The other case is similar,

$$\begin{aligned} \min_x \quad & \max_{i=1,\ldots,n} f_i(x) \\ s.t. \quad & f_{i3}(x) < 0 \end{aligned}$$

is also a quasi-convex function optimization.

The above two optimization problems can be solved by a sequence of SOCP feasibility problems [19].

B. Triangulation with Uncertain Data

Given a fixed parameter γ, the constraint $d(\cdot, \cdot) \leq \gamma$ can be converted to be a standard SOCP problem:

$$\|Ax + b\|_2 \leq \gamma(c^T x + d)$$

where$A = \begin{bmatrix} x_i p^{3T} - p^{1T} \\ y_i p^{3T} - p^{2T} \end{bmatrix}$, $b = 0$, $c = p^{3T}$, $d = 0$.

The theorem can be formulated as following specific form with respect to triangulation problem. The triangulation problem comply with the following assumption:

- the image point $(x_i, y_i)^T$ are perturbed by internal data.
- the right hand side perturbation set Z with respect to γ is given by $\gamma \in [0, +\infty]$, which is a finite list of linear inequality.
- the uncertainty is side-wise, i.e. the image point perturbation is independent of γ.

Theorem 2. Under above assumptions on the perturbation set Z, the triangulation with uncertain data is equivalent to the following explicit system of conic quadratic and linear constraints in variables y, z:

$$\begin{aligned} \|z\|_2 &\leq \gamma p^{3T} x \\ z_i &\geq |(A^n x + b^n)_i| + \delta_i + \sum_{j=1}^{n} |\delta_{ij} x|, 1 \leq i \leq k \end{aligned} \quad (4)$$

where δ_{ij} and δ_i are the bound of A_{ij} and b_i, respectively.

Proof. Under above assumptions on the perturbation set Z, the triangulation with uncertain data is equivalent to the following explicit system of conic quadratic and linear constraints in variables y, z, τ:

$$\begin{aligned} \tau &\leq \gamma p^{3T} x \\ z_i &\geq |(A^n x + b^n)_i| + \delta_i + \sum_{j=1}^{n} |\delta_{ij} x|, 1 \leq i \leq k \\ \|z\|_2 &\leq \tau \end{aligned}$$

The first constraint and the third one can be amalgamated into a SOCP problem: $\|z\|_2 \leq \gamma p^{3T} x$. This situation is a specific example of theorem 1.

According the results of Theorem 1 and Theorem 2, we design the Robust Optimization Triangulation Algorithm to solve the triangulation problems with uncertain data.

Algorithm 1 Robust Optimization Triangulation Algorithm

Require:

Corresponding Image Points $\vec{x_i}$, $\vec{x_i'}$;

Camera Matrix P_i;

Ensure:

1: Solving SOCP problems of variables z, x

$$\|z\|_2 \leq \gamma p^{3T} x$$

If the above problem is feasible, go to next step, else exit;

2: Verify if the above SOCP problem complies with the following robust constraints:

$$z_i \geq |(A^n x + b^n)_i| + \delta_i + \sum_{j=1}^{n} |\delta_{ij} x|, \quad 1 \leq i \leq k$$

3: If feasible, return x, else no robust solution. Goto Step 1 to reset the parameters γ.

V. Algorithms and Experimental Results

In this section, robust optimization triangulation algorithm with uncertain data is tested on the simulation data and the real images. All the algorithms have been implemented in the MATLAB environment using the convex solver SeDuMi [2].

We first present the robust optimization triangulation algorithm. In this robust optimization triangulation algorithm, the original data A^n and b^n are not known. So they are replaced by perturbational data A and b. The synthetic data experimental results show that there is no more distinct effect on their performance.

A. Synthetic Data Experiments

We first evaluate the robustness of our algorithm against noise by numerical simulation. Our data is generated by creating 100 random space points and then projecting to 50 images through 50 different views. To find the relationship between the noise and the error, the noise is attached to the space points instead of the image points. The space points are corrupted with i.i.d. Gaussian noise with zero mean and variance range from zero to 0.10 meter with step 0.01 meter. The Robust Optimization Triangulation Algorithm is employed to compute the 3D space points' coordinate. The estimated points are reprojected on the images again. The reprojection errors of all the image points were computed to find the relationship between the reprojection image error and noisy data.

We employed the RMS error and the L_∞ error to evaluate the performance of algorithms [24]. The RMS error and L_∞ error are defined as follows:

$$RMS \;\; error = \left[\frac{1}{2m} \sum_{i=1}^{m} \epsilon_i^2 \right]^{1/2} \tag{5}$$

$$L_\infty \;\; error = \max_{i=1,\ldots,m} |\epsilon_i| \tag{6}$$

where ϵ_i is the error of the ith pair of corresponding image points.

Fig. 2 shows the 100 space points (red star) and corresponding perturbed points (blue diamond), respectively. Fig. 3 shows the 50 image points (red star) and corresponding reprojected points (blue triangle), respectively. Fig. 4 shows the $Linf\_error$ and RMS_error for different image noise level. We can see that the RMS_error increase almost linearly as the noise level grows. The $Linf\_error$ range from zero to 0.33 meters when the noise changes from zero to 0.10 meter. The results of the synthetic data are proof that the L_∞ norm method is sensitive to the noise and outliers.

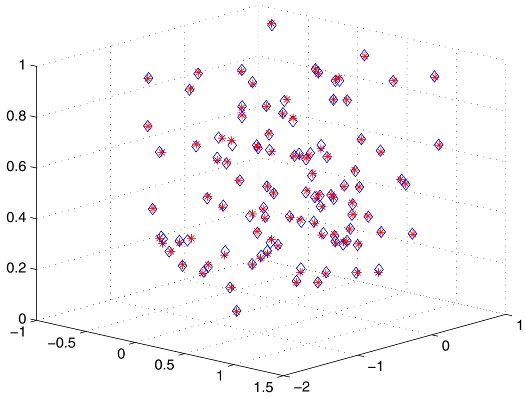

Fig. 2. space points and corresponding perturbed points

B. Real Data Experiments

To evaluate the performance of our algorithm against real data, we have made extensive use of a publicly available dinosaur sequences with given feature correspondences. The image set is a turntable sequence of a dinosaur, containing 36 images and in total 328 image correspondences with lots of occlusions. We reconstructed 4983 space points according the image correspondences.

Fig. 5 shows 2 images of the 36 images. We also found that if the threshold $Bound$ for the space points and the number of Maximum or Minimum point n were given, some outliers can be removed from the original data set. Algorithm 2

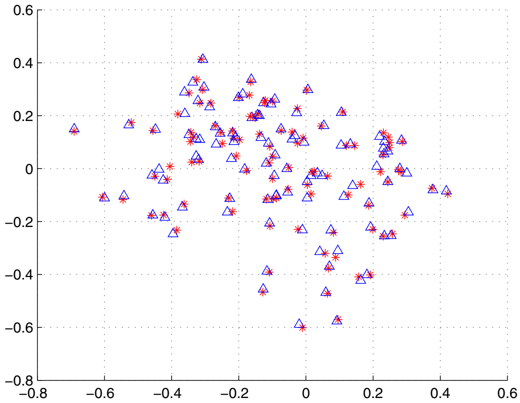

Fig. 3. Image points and corresponding perturbed points

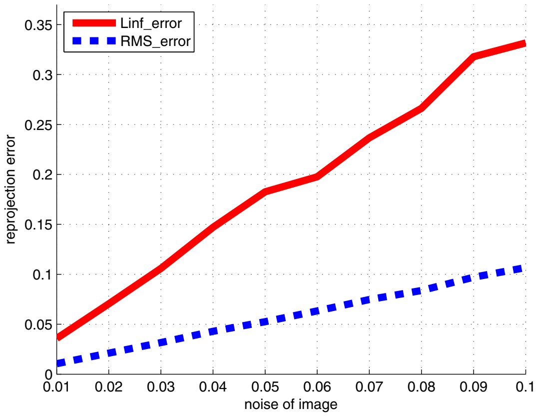

Fig. 4. Linf_error and RMS_error for different image noise level

shows the pseudo code of Robust Optimization Triangulation Outlier Removal Algorithm.

Fig. 6 and Fig. 7(only 500 points has been plotted) shows the results of before and after removing the $1157th$ point from the estimated points. In the upper left side of the left figure of Fig. 6, there is an isolated point, i.e., the $1157th$ space point. If the $Bound$ has been set a smaller value, the $1157th$ space point will be removed from the reconstructed points set. Fig. 8 and Fig. 9(only 500 points has been plotted) shows the results of before and after removing the $4582th$ point (in the lower right corner) from the estimated points. We can see that the $1157th$ and $4582th$ point have been removed from the data set. The difficulty of this method is how to set the parameters like $Bound$ since we can not know which one is the best one.

VI. Conclusion

In this paper, Robust Optimization are employed to solve the triangulation problems with uncertain(perturbational) data. The proposed approach has shown good performance with synthetic data and real image data. The advantage of this method is guarantee of the global optimality, that is, the

Fig. 5. 2 images of Dinosaur Sequence

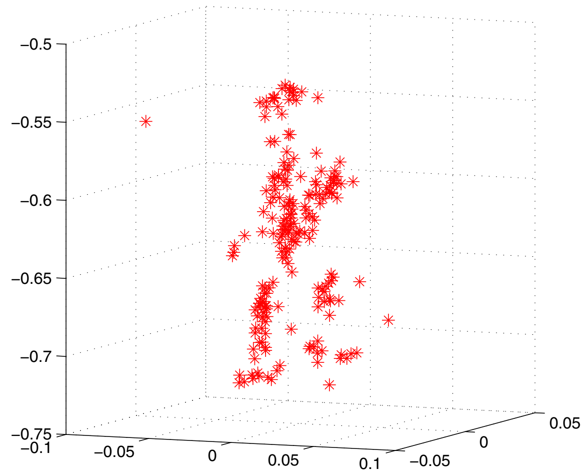

Fig. 6. Outlier:1157th point has not been removed

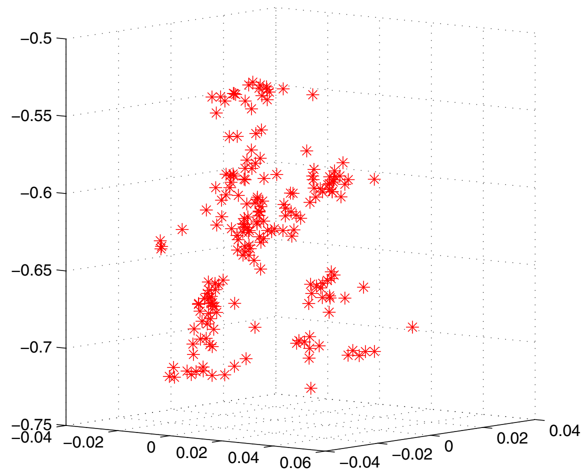

Fig. 7. Outlier:1157th point has been removed

Algorithm 2 Robust Optimization Triangulation Outlier Removal Algorithm

Require:

outlier removal bound: $Bound$;

the number of Maximum or Minimum point: n;

Ensure:

1: the reconstructed space point coordinate $X_{estimated}$;

2: Find the index set $Index$ according the given number n;

3: Find the corresponding image point according the bound $Bound$. If satisfied, go to next step, else exit;

4: Solving the corresponding robust optimization problem. If infeasible, the current point is outlier, else the current point is inlier.

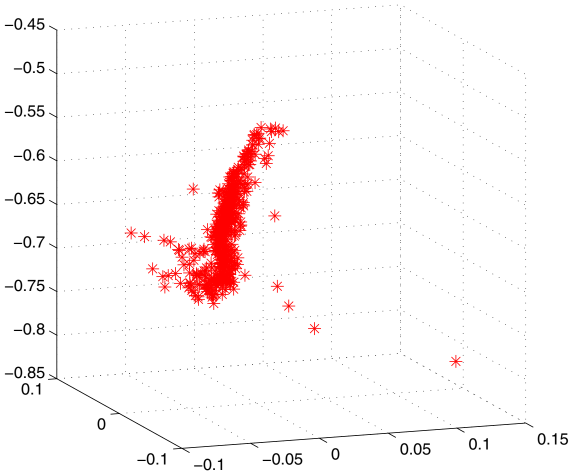

Fig. 8. Outlier:4582th point has not been removed

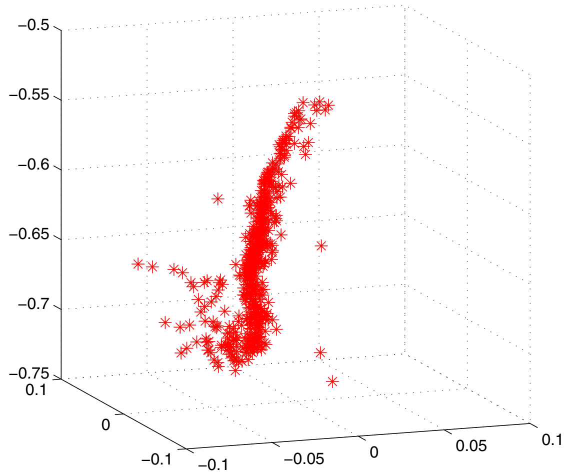

Fig. 9. Outlier:4582th point has been removed

best possible solution in the worst case can be found. In the future, the Robust Optimization algorithm can be applied to other multiview geometry problems and more robust outlier removal algorithm will be considered.

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