



Universität Hamburg

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Department of Informatics



# Introduction to Robotics

## Lecture 11

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University of Hamburg  
Faculty of Mathematics, Informatics and Natural Sciences  
Department of Informatics

**Technical Aspects of Multimodal Systems**

June 28, 2018

Introduction

Coordinate systems

Kinematic Equations

Robot Description

Inverse Kinematics for Manipulators

Differential motion with homogeneous transformations

Jacobian

Trajectory planning

Trajectory generation

Dynamics

Principles of Walking

Robot Control

Task-Level Programming and Trajectory Generation



# Outline (cont.)

## Task-level Programming and Path Planning

Work space to Configuration Space

C-obstacles

Partition Representation of the C-Space

## Task-level Programming and Path Planning

Architectures of Sensor-based Intelligent Systems

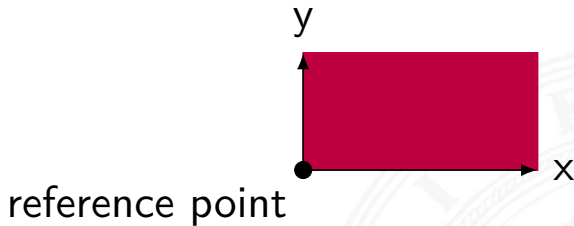
Summary

Conclusion and Outlook



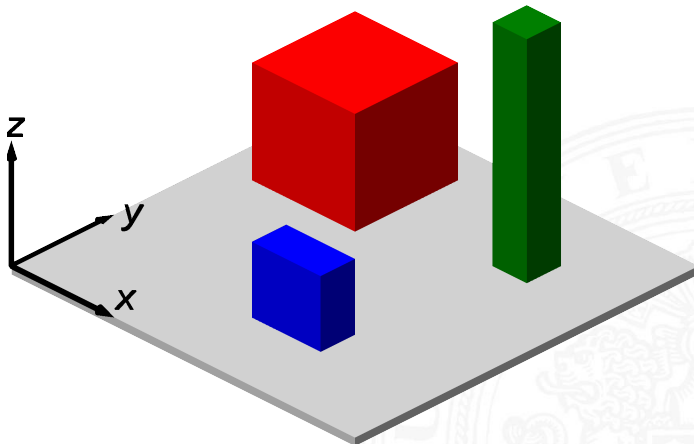


**Robot** Single reference point with physical attributes





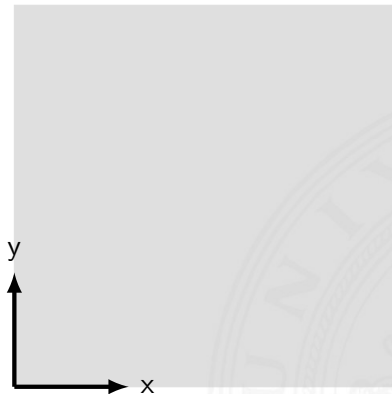
**Work space** The cartesian space of the environment





# Task-level Programming – Basics

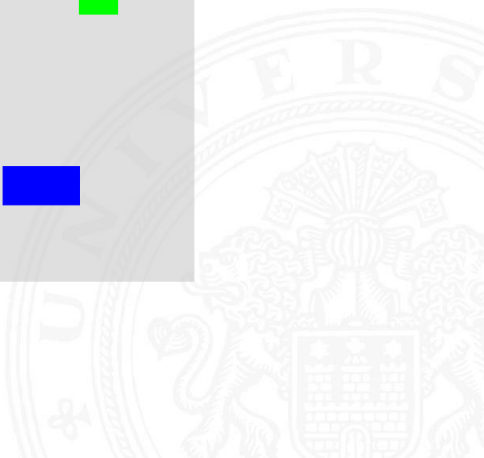
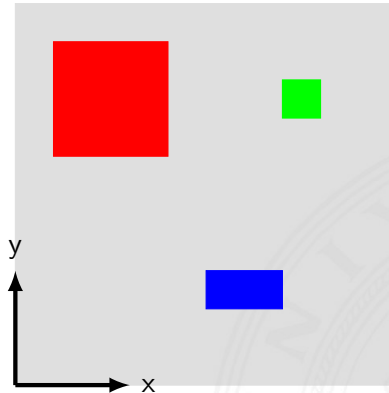
Configuration space  $C$  Set of all possible configurations





# Task-level Programming – Basics

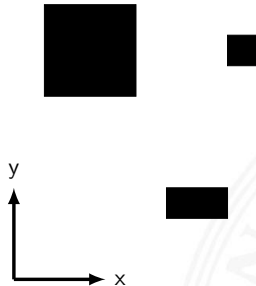
Obstacles in work space C-Obstacles in configuration space





# Task-level Programming – Basics

Obstacle space  $C_{\text{obstacle}}$  Union of C-Obstacles

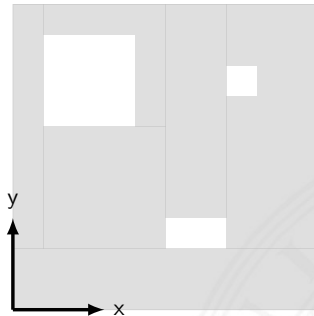






# Task-level Programming – Basics

Free space  $C_{\text{free}}$  the complement of Obstacle space



**Robot** Single reference point with physical attributes

**Work space** The cartesian space of the environment

**Configuration space  $C$**  Set of all possible configurations

**Obstacles in work space**  $C$ -Obstacles in configuration space

**Obstacle space  $C_{\text{obstacle}}$**  Union of  $C$ -Obstacles

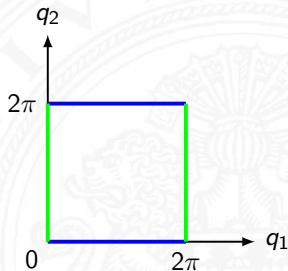
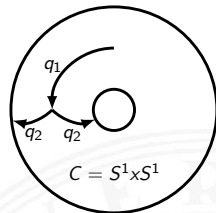
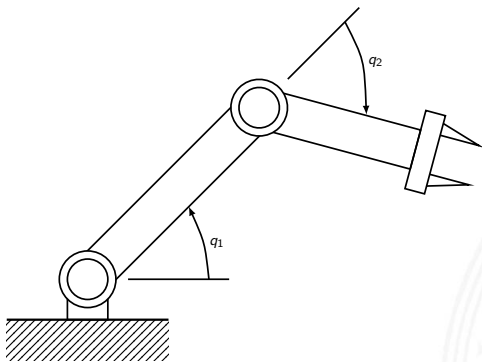
**Free space  $C_{\text{free}}$**  the complement of Obstacle space

**Path-planning** for Work-/Configuration-Space

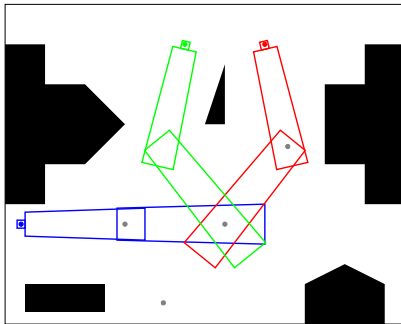
Search for a path for the reference point of the artifact in the free space.

Configurations of the artifact in free space have no intersection with obstacles

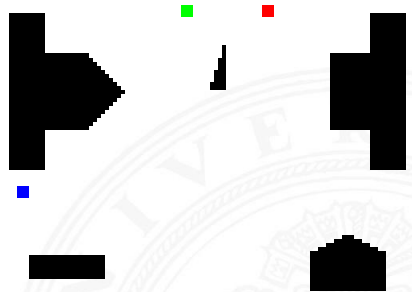
# Work Space to Configuration Space – Illustration



# Work Space to Configuration Space – Example

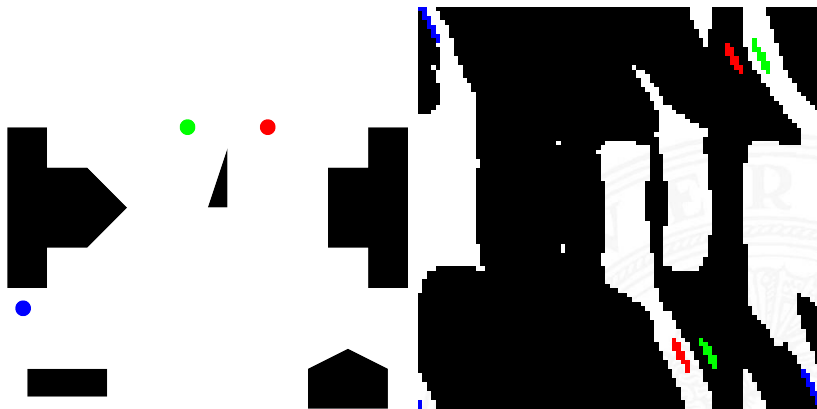


Workspace scheme with start and goal positions



Discretized workspace  
 $x_{scale} = 100, y_{scale} = 80$

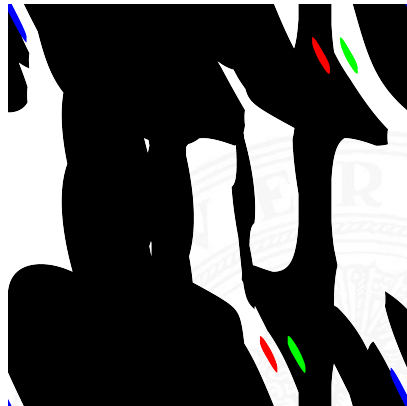
# Work Space to Configuration Space – Example



Discretized workspace  $x^{scale} = 2000$ ,  
 $y^{scale} = 1600$

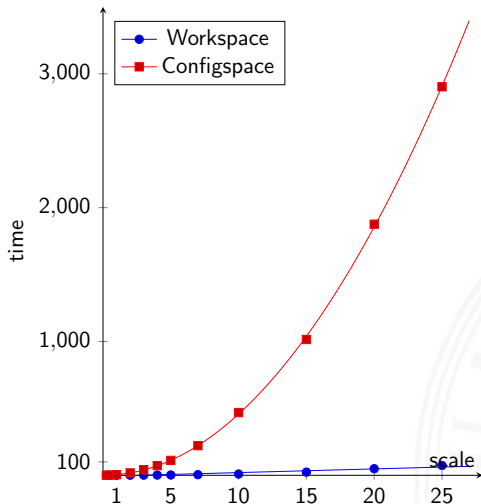
Discretized configuration space  
 $q_1^{scale} = 90$ ,  $q_2^{scale} = 90$

# Work Space to Configuration Space – Example



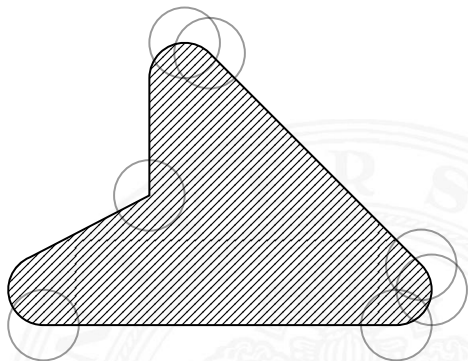
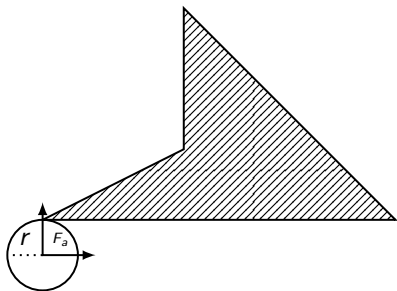
Discretized configuration space  
 $q_1^{scale} = 3600, q_2^{scale} = 3600$

# Work Space to Configuration Space – Complexity



- ▶ Python
- ▶ Brute forward kinematics
- ▶ using polygon collisions
  - ▶ shapely library
- ▶ 56 cpus
  - ▶ Intel® Xeon® E5-2690 v4 (2.60GHz)

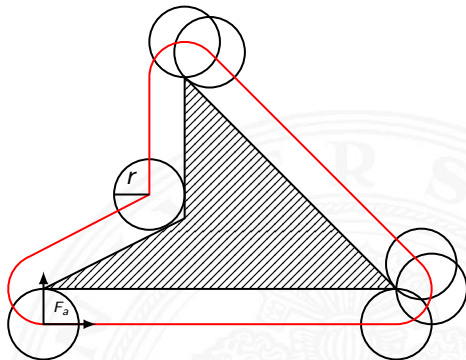
# C-Obstacle for a circular artifact



Obstacle & artifact (radius  $r$ ) Expanded  
C-Obstacle



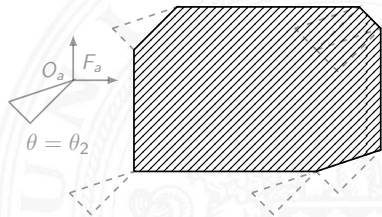
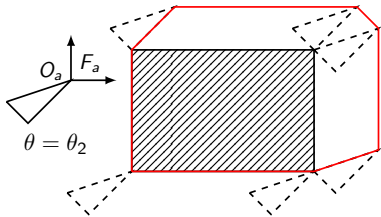
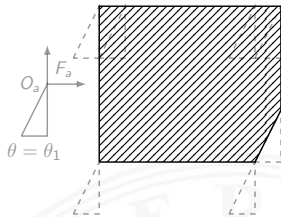
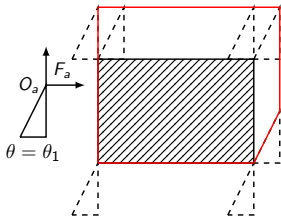
# C-Obstacle for a circular artifact



Obstacle & artifact (radius  $r$ )

Path of minimal distance to obstacle

# C-Obstacle for Polygons



Obstacle & polygon artifact with  $\theta = \theta_1 \vee \theta_2$ ; minimum distance to obstacle.

A C-Obstacle of a fixed, convex obstacle with respect to a moving convex robot (part) may be theoretically represented as the Minkowski Sum of the corresponding objects.

$C_O(H)$  is the C-obstacle of a fixed convex polyhedra  $H$ , with respect to the (moving) convex object  $O$ .

**Minkowski-Sum (Minkowski-Difference) of  $H$  and  $O$  ( $H$  and  $-O$ )**

$$C_O(H) = H \ominus O = H \oplus (\ominus O)$$

where

$$H \ominus O := \{h - o \mid h \in H \wedge o \in O\}$$

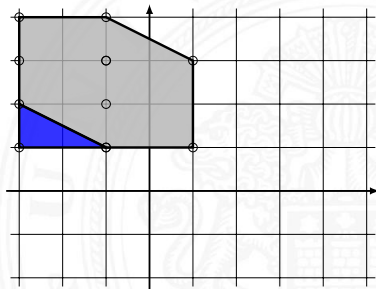
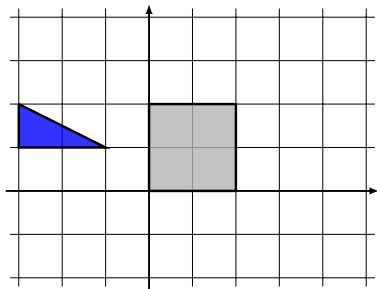
# Minkowski Sum & Difference – 2D Example

$$A = \{(0, 0), (2, 0), (2, 2), (0, 2)\} \quad B = \{(-1, 1), (-3, 2), (-3, 1)\}$$

$$A \oplus B = \{(-1, 1), (-3, 2), (-3, 1), (1, 1), (-1, 2), (-1, 1), \\ (1, 3), (-1, 4), (-1, 3), (-1, 3), (-3, 4), (-3, 3)\}$$

The convex hull (eliminating duplicates & inner points)

$$\text{conv}\{A \oplus B\} = \{(-3, 1), (1, 1), (1, 3), (-1, 4), (-3, 4)\}$$



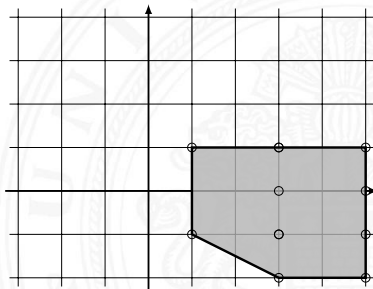
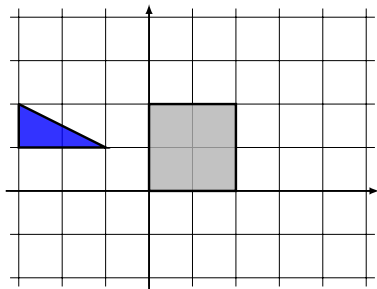
# Minkowski Sum & Difference – 2D Example (cont.)

$$A = \{(0, 0), (2, 0), (2, 2), (0, 2)\} \quad B = \{(-1, 1), (-3, 2), (-3, 1)\}$$

$$A \ominus B = \{(1, -1), (3, -2), (3, -1), (3, -1), (5, -2), \\ (5, -1), (3, 1), (5, 0), (5, 1), (1, 1), (3, 0), (3, 1)\}$$

The convex hull (eliminating duplicates & inner points)

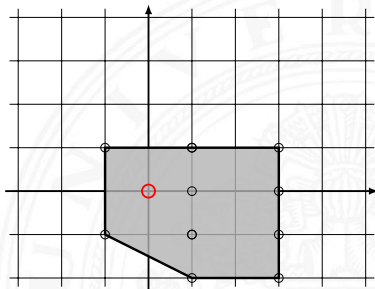
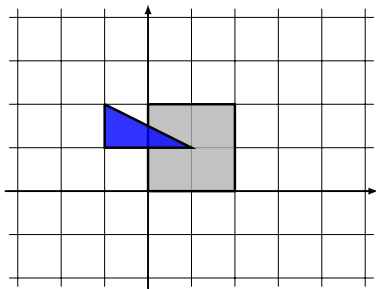
$$\text{conv}\{A \ominus B\} = \{(1, -1), (3, -2), (5, -2), (5, 1), (1, 1)\}$$



# Minkowski Sum & Difference – 2D Example (cont.)

## Collision detection

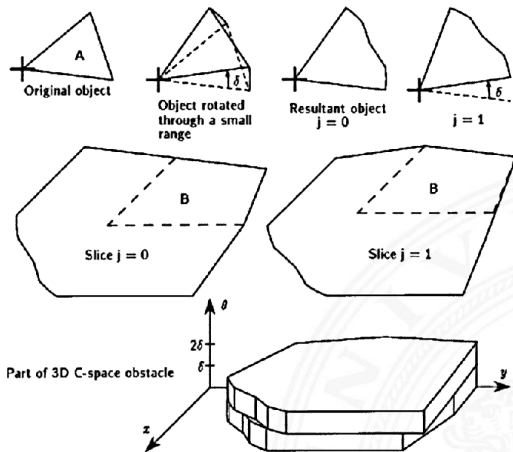
Two objects are colliding, if their Minkowski difference contains the origin of the coordinate frame.



There is an interactive applet on the web:

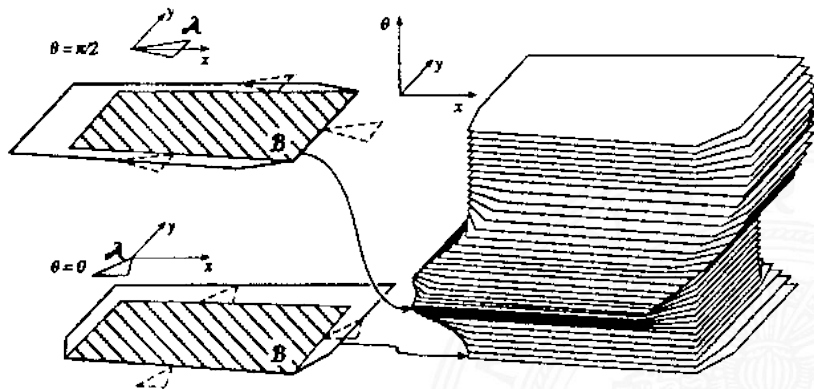
<http://www.cut-the-knot.org/Curriculum/Geometry/PolyAddition.shtml>

# C-Obstacles for 2-D translation and 1-D rotation



Represent rotational configuration of the C-obstacle as slice for each  $\theta$  configuration of the robot.

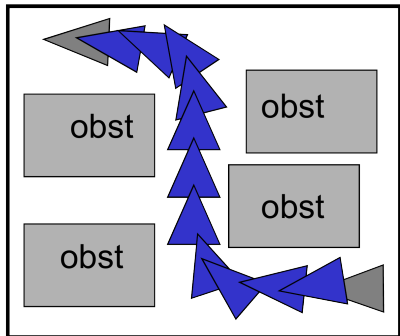
# C-Obstacles for 2-D translation and 1-D rotation (cont.)



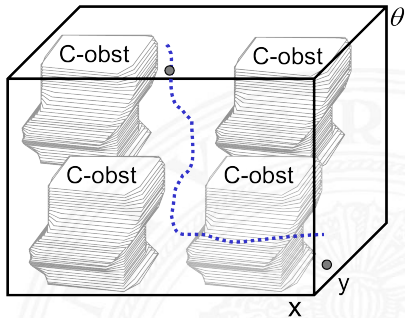
The configuration space for a  $k$ -DOF robot is a  $k$ -Dimensional coordinate system.



# C-Obstacles for 2-D translation and 1-D rotation (cont.)

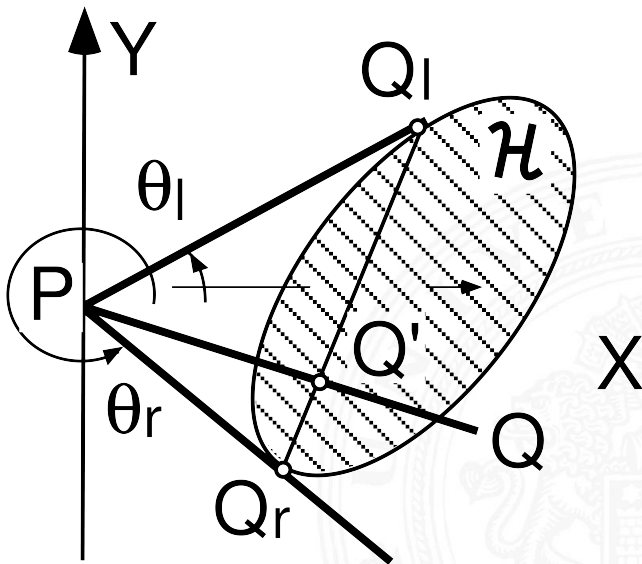


Work space  $(x, y)$

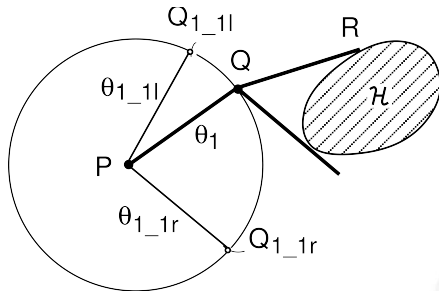


Configuration space  $(x, y, \theta)$

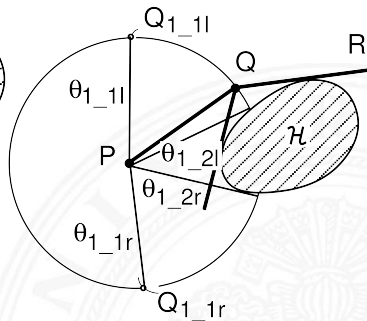
# C-obstacles of a pole



# C-obstacles of a 2-DOF Chain of Poles

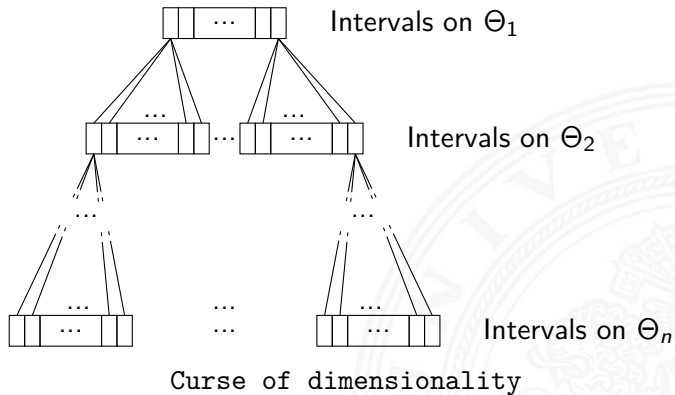


(1)

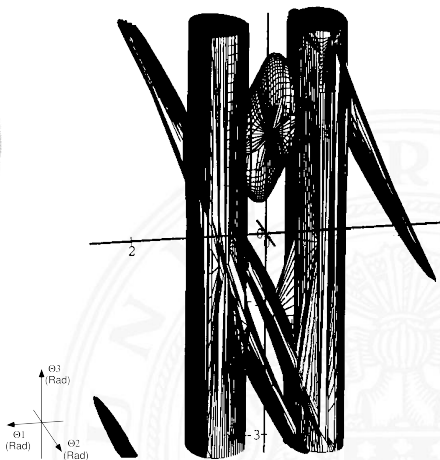
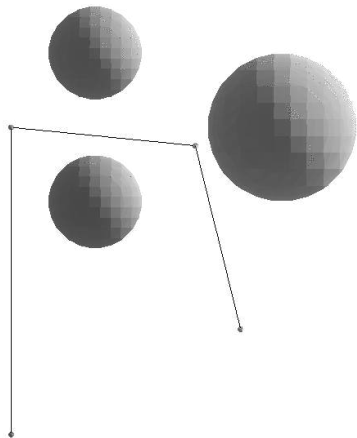


(2)

# Tree-structure for Configuration Space partitioning



# Configuration Space of a 3-DOF Chain of Poles





The free space is partitioned into cells using

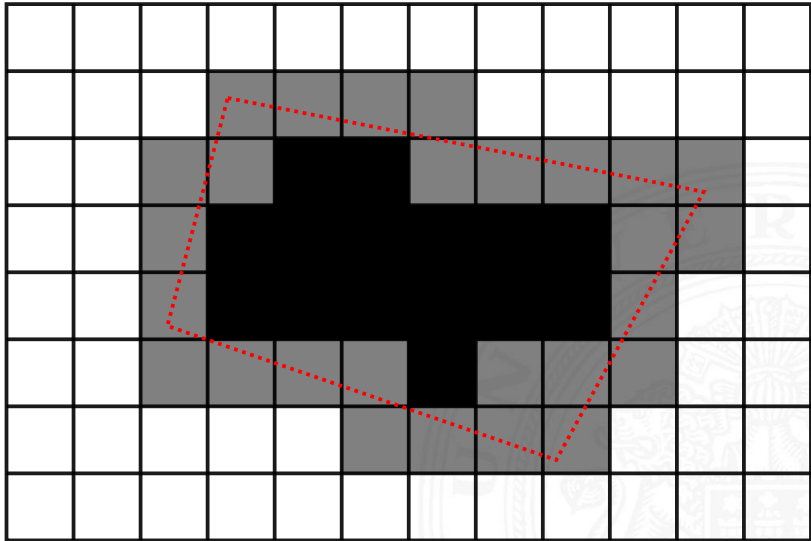
- ▶ Geometrical partition
  - ▶ uniform cubes
  - ▶ a hierarchical tree-structure (Quad-tree, Oct-tree, etc.)
  - ▶ slices and scanlines
  - ▶ bubbles of variable size

The union of the non-overlapping cells is part of the free space.  
Neighborhood graphs represent the connectivity of free space.

- ▶ Topological partition
  - ▶ overlapping generalized cones
  - ▶ critical points of the C-obstacle connection graph

The union of the overlapping cells is equal to the free space.

# Squares-Partitioning of Configuration Space



Resulting bitmap of configuration space using squares partitioning

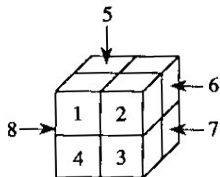
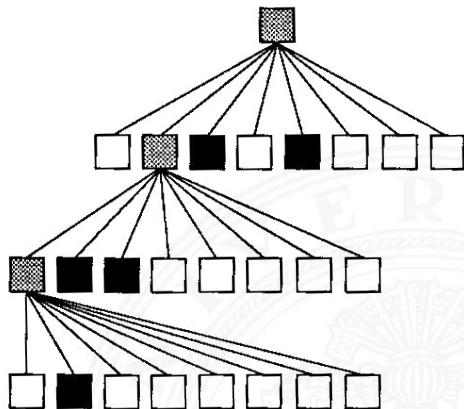
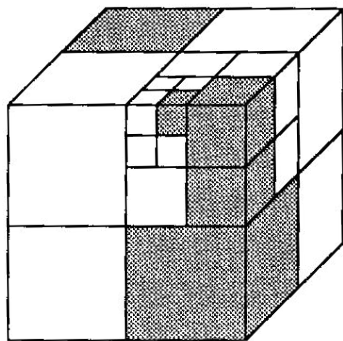
# Squares-Partitioning of Configuration Space (cont.)



Bitmap of configuration space

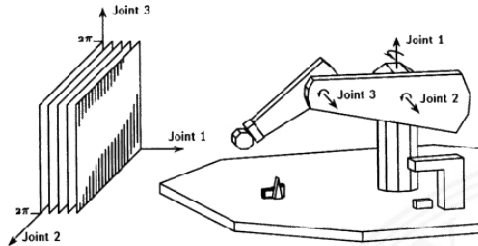


# Partitioning of the configuration space using Octrees



 EMPTY cell     MIXED cell     FULL cell

# Partitioning of the configuration space using Slices



## Complexity regarding the transformation of the C-obstacles

$$r^{d-1}f(m)$$

where  $r$ : the number of discretization steps for each DOF,

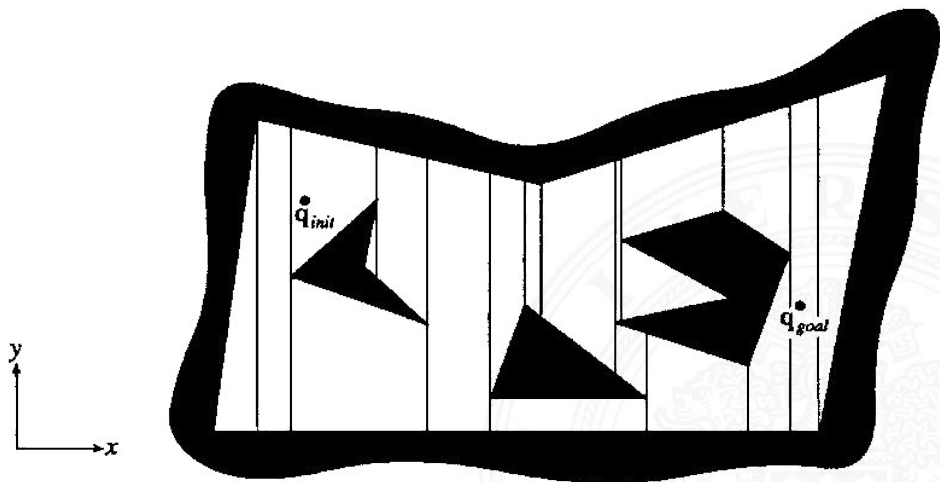
$d$ : DOF of the robot arm

$f(m)$ : the computing time of one slice

$m$ : the number of edges of all obstacles

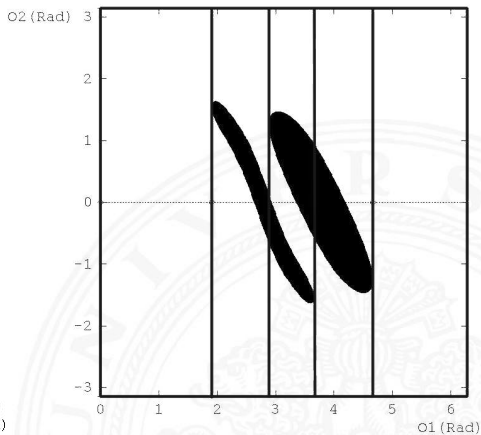
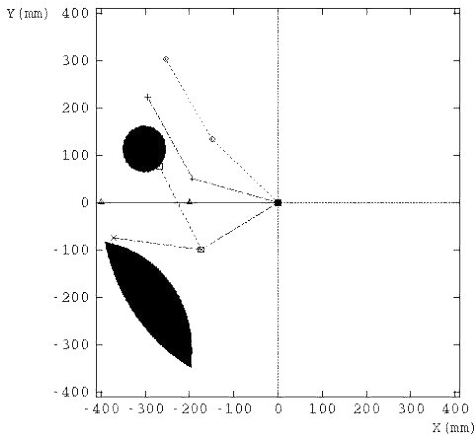


# Exact Partition of Configuration Space



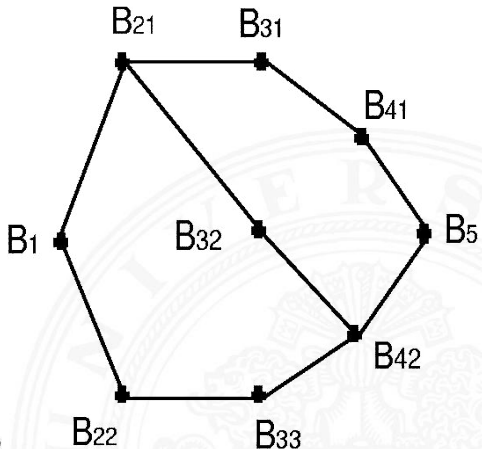
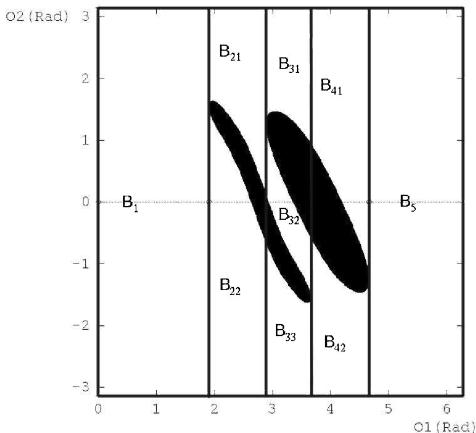
Trapezoidal partitioning of the configuration space

# Exact Partition of Configuration Space (cont.)



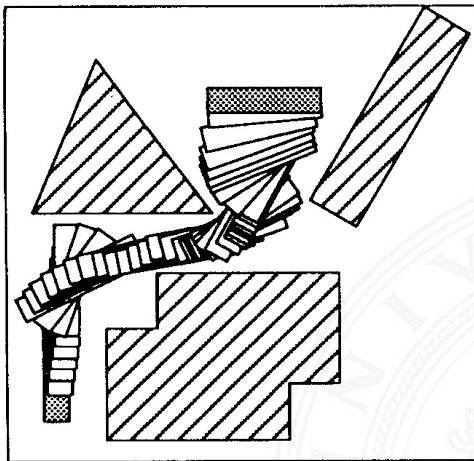
Cylindrical partitioning using critical points

# Exact Partition of Configuration Space (cont.)



Cylindrical partitioning and connectivity graph

# Planning Results



[12]

Serial computing: 3-DOF C-space

Massive-parallel computing: up to 6-DOF C-Space



## Advantages:

- ▶ Complete in case of sufficient resolution
- ▶ Global overview

## Disadvantages:

- ▶ High demand for RAM
  - ▶ Curse of Dimensionality
- ▶ Complex to implement
- ▶ Practically implementable only for few degrees of freedom



Path planning without explicit representation of free space?



next Lecture!





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