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Introduction to Robotics

Assignment #1

Due: 20.04.2021, 23:59

Task 1.1 (8 points) Pyramid: A pyramid (square base $AB = BC = CD = DA = 42\,mm$; plumb-line $ME = 12\,mm$, with vertex E located at the top and point M located at the center of the base) is held by a robot so that its square base $ABCD$ is located in the xy-plane of a cartesian world coordinate frame M_{xyz}, with point M at its origin, the edges AB and CD parallel to the x-axis and the edged BC and AD parallel to the y-axis. Attached to the pyramid is an object coordinate frame M_{uvw}, which initially coincides with M_{xyz}. Write down the general transformation matrix for each rotation.

1.1.1 (4 points): Determine the locations of the vertices A through E, after the following sequence of rotations has been performed by the robot:

1. Rotation by $\psi = 50°$ around M_w
2. Rotation by $\varphi = -35°$ around M_u
3. Rotation by $\theta = 340°$ around M_v

1.1.2 (4 points): Same sequence of rotations, but using the rotation axes M_z, M_x and M_y instead.

Task 1.2 (6 points) Homogeneous transformations: Given are three frames A, B and C as well as the following two homogeneous transformations:

$$ {}^{A}T_{B} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 & 1 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

and

$$ {}^{B}T_{C} = \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 & 2 \\ 1/2 & \sqrt{3}/2 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

1.2.1 (3 points): Can the interpretation of the transformation ${}^{A}T_{C}$ be considered to be unambiguous? Explain your answer.

1.2.2 (3 points): Visualize the three coordinate systems with a tool of your choice.

Task 1.3 (6 points) Euler angles:

1.3.1 (4 points): Give four examples of Euler angle combinations (φ, θ, ψ) and interpret their geometric meaning using natural language.
"This is a rotation around x by φ" is not sufficient. Explain the properties of the transformation with respect to some real objects. For example, a plane, a toy or a humanoid robot and so on.

1.3.2 (2 points): There are 12 possible sequences of rotations with Euler-angles around the axes (see slide 29). Explain why there are exactly 12!