



# Introduction to Robotics

## Lecture 3

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**Technical Aspects of Multimodal Systems**

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Introduction

Spatial Description and Transformations

Forward Kinematics

Robot Description

- Recapitulation of DH-Parameter

- URDF

Inverse Kinematics for Manipulators

Velocity of rigid body

Jacobian

Trajectory planning

Trajectory generation

Dynamics

Robot Control

Task-Level Programming and Trajectory Generation





# Outline (cont.)

Robot Description

Introduction to Robotics

Task-level Programming and Path Planning

Task-level Programming and Path Planning

Architectures of Sensor-based Intelligent Systems

Summary

Conclusion and Outlook



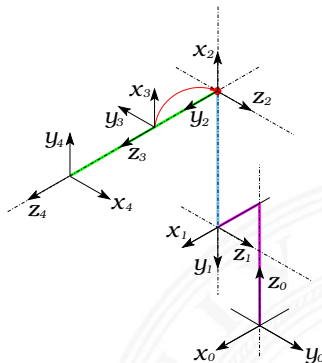
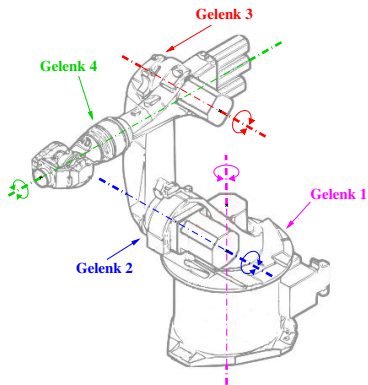


- ▶ universal minimal robot description
- ▶ based on frame transformations
- ▶ **four** parameters per frame transformation
- ▶ serial chain of transformations
- ▶ unique description of  $T_6$

## Drawbacks

- ▶ ambiguous convention
- ▶ only kinematic chain described
- ▶ missing information on geometry, physical constraints, dynamics, collisions, inertia, sensors, . . .

# Definition of joint coordinate systems



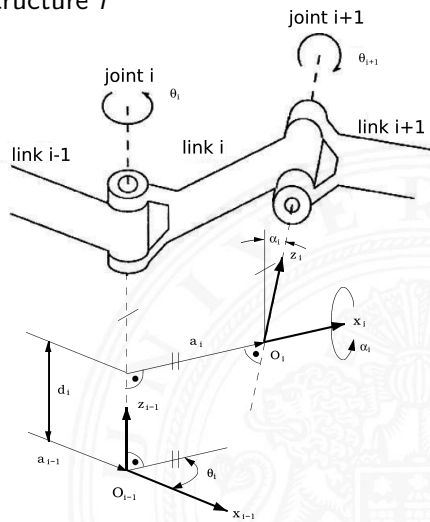
- ▶  $CS_0$  is the stationary origin at the base of the manipulator
- ▶ axis  $z_{i-1}$  is set along the axis of motion of the  $i^{th}$  joint
- ▶ axis  $x_i$  is the common normal of  $z_{i-1} \times z_i$
- ▶ axis  $y_i$  concludes a right-handed coordinate system

# Parameters for description of two arbitrary links

Two parameters for the description of the link structure  $i$

- ▶  $a_i$ : shortest distance between the  $z_{i-1}$ -axis and the  $z_i$ -axis
- ▶  $\alpha_i$ : rotation angle around the  $x_i$ -axis, which aligns the  $z_{i-1}$ -axis to the  $z_i$ -axis

$a_i$  and  $\alpha_i$  are constant values due to construction



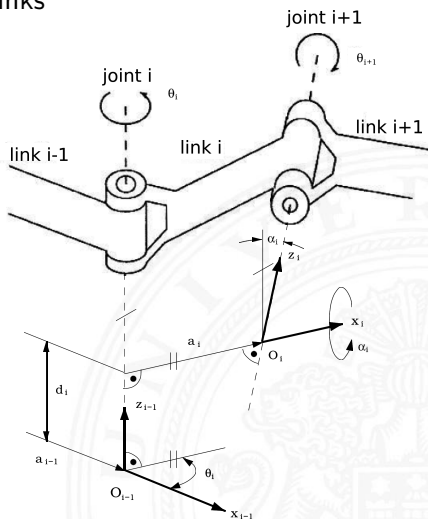
# Parameters for description of two arbitrary links (cont.)

Two for relative distance and angle of adjacent links

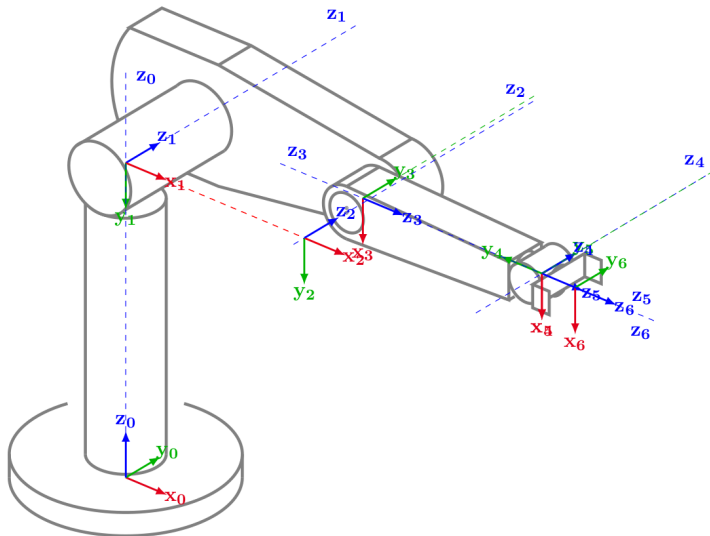
- ▶  $d_i$ : distance origin  $O_{i-1}$  of the  $(i-1)^{\text{st}}$  CS to intersection of  $z_{i-1}$ -axis with  $x_i$ -axis
- ▶  $\theta_i$ : joint angle around  $z_{i-1}$ -axis to align  $x_{i-1}$ - parallel to  $x_i$ -axis into  $x_{i-1}, y_{i-1}$ -plane

$\theta_i$  and  $d_i$  are variable

- ▶ rotational:  $\theta_i$  variable,  $d_i$  fixed
- ▶ translational:  $d_i$  variable,  $\theta_i$  fixed



# Example featuring PUMA 560







## DH parameters of PUMA 560

Joint	$a_i$	$\alpha_i$	$d_i$	$\theta_i$
1	0	$-\frac{\pi}{2}$	$d_1$	$\theta_1^*$
2	$a_2$	0	0	$\theta_2^*$
3	$a_3$	$\frac{\pi}{2}$	$d_3$	$\theta_3^*$
4	0	$-\frac{\pi}{2}$	$d_4$	$\theta_4^*$
5	0	$\frac{\pi}{2}$	0	$\theta_5^*$
6	0	0	$d_6$	$\theta_6^*$

In order to transfer the manipulator-endpoint into the base coordinate system,  $T_6$  is calculated as follows:



## Documentation

<http://wiki.ros.org/urdf>

<http://wiki.ros.org/urdf/XML>

<http://wiki.ros.org/urdf/Tutorials>

- ▶ robot description format used in ROS<sup>19</sup>
- ▶ hierarchical description of components
- ▶ XML format representing robot model
  - ▶ kinematics and dynamics
  - ▶ visual
  - ▶ collision
  - ▶ configuration

---

<sup>19</sup><http://ros.org>

▶ 1<sup>st</sup>-level structure

```
<robot name="samplerobot">  
</robot>
```

▶ 2<sup>nd</sup>-level structure

`link`, `joints`, `sensors`, `transmissions`, `gazebo`, `model_state`

▶ 3<sup>rd</sup>-level structure

`visual`, `inertia`, `collision`, `origin`, `parent`, ...

▶ 4<sup>th</sup>-level structure

⋮



- ▶ Filename: robotname.urdf
- ▶ XML prolog:

```
<?xml version="1.0" encoding="utf-8"?>
```

- ▶ XML element types

```
<tag attribute="value"/>
```

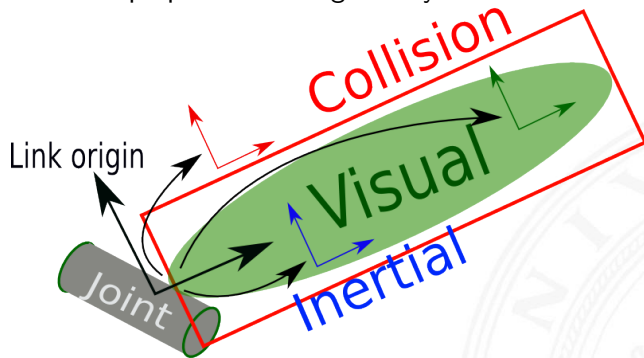
```
<tag attribute="value">  
text or element(s)  
</tag>
```

- ▶ XML comments

```
<!-- Comments are placed within these tags -->
```



Link describes geometrical properties of a rigid body.



20



```
<link name="sample_link">
<!-- describes the mass and inertial properties of
the link -->
<inertial/>

<!-- describes the visual appearance of the link.
can be described using geometric primitives or
meshes -->
<visual/>

<!-- describes the collision space of the link.
is described like the visual appearance -->
<collision/>
</link>
```

<sup>20</sup><http://wiki.ros.org/urdf/XML/link>



## Geometric primitives for describing visual appearance of the link

```
<link name="base_link">
  <visual>
    <origin xyz="0 0 0.01" rpy="0 0 0"/>
    <geometry>
      <box size="0.2 0.2 0.02"/>
    </geometry>
    <material name="cyan">
      <color rgba="0 1.0 1.0 1.0"/>
    </material>
  </visual>
</link>
```

- ▶ Geometric primitives: `<box>`, `<cylinder>`, `<sphere>`
- ▶ Materials: `<color>`, `<texture>`



## 3D meshes for describing visual appearance of the link

```
<link name="base_link">
  <visual>
    <origin xyz="0 0 0.01" rpy="0 0 0"/>
    <geometry>
      <mesh filename="meshes/base_link.dae"
    </geometry>
  </visual>
  <collision>
    <origin xyz="0 0 0.01" rpy="0 0 0"/>
    <geometry>
      <cylinder radius="1" length="0.5"/>
    </geometry>
  </collision>
</link>
```

- ▶ the `<collision>` element can be simpler from the `<visual>` in order to reduce computation time





Parameters describing the physical properties of the link

```
<link name="base_link">
  <inertial>
    <origin xyz="0 0 0" rpy="0 0 0"/>
    <mass value="1">
      <inertia ixx="100" ixy="0" ixz="0"
              iyy="100" iyz="0" izz="100" />
    </inertial>
  </link>
```

- ▶ center of gravity <origin xyz>
- ▶ object mass <mass value>
- ▶ inertia tensor <intertia>



Inertia tensor describes the distribution of mass of the link

- ▶ orientation and position of the inertia CS described by `<origin>` tag

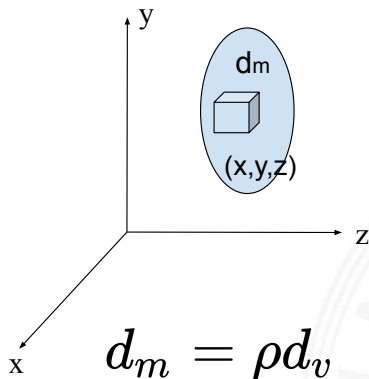


$$A_I = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix}$$

- ▶ diagonal values describe main inertial axes `ixx`, `iyy`, `izz`
- ▶ `ixy`, `ixz`, `iyz` are 0 for symmetric primitives
- ▶ rotations around largest and smallest inertial axis are most stable

- ▶ moments of inertia:

$$I_{xx} = \int (y^2 + z^2) dm \quad I_{yy} = \int (x^2 + z^2) dm \quad I_{zz} = \int (x^2 + y^2) dm$$

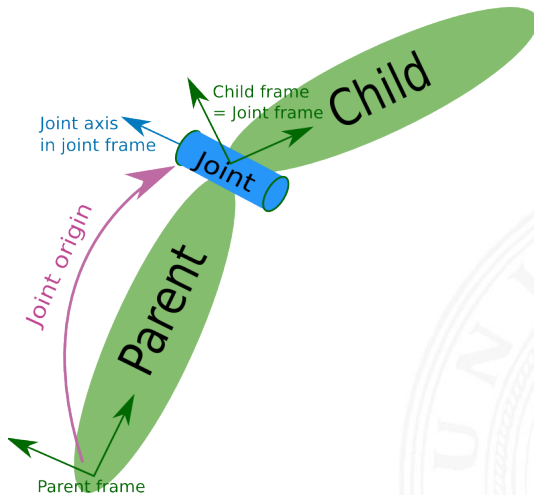


- ▶ Products of Inertia:

$$I_{xy} = I_{yx} = \int xy dm \quad I_{zy} = I_{yz} = \int yz dm \quad I_{xz} = I_{zx} = \int zy dm$$



Joint describes geometrical connections of two links.





```
<joint name="base_link_to_cyl" type="revolute">
  <!-- describes joint position and orientation -->
  <origin xyz="0 0 0.07" rpy="0 0 0"/>

  <!-- describes the related links -->
  <parent link="base_link"/>
  <child link="base_cyl"/>

  <!-- describes the axis of rotation-->
  <axis xyz="0 0 1"/>

  <!-- describes the joint limits-->
  <limit velocity="1.5707963267"
          lower="-3.1415926535" upper="3.1415926535"/>
</joint>
```

<sup>21</sup><http://wiki.ros.org/urdf/XML/joint>



`type` `revolute`, `continuous`, `prismatic`, `fixed`, `floating`, `planar`

`parent_link` link which the joint is connected to

`child_link` link which is connected to the joint

`axis` joint axis relative to the joint CS. Represented using a normalized vector

`limit` joint limits for motion (`lower`, `upper`), `velocity` and `effort`

`dynamics` damping, friction

`calibration` rising, falling

`mimic` joint, multiplier, offset

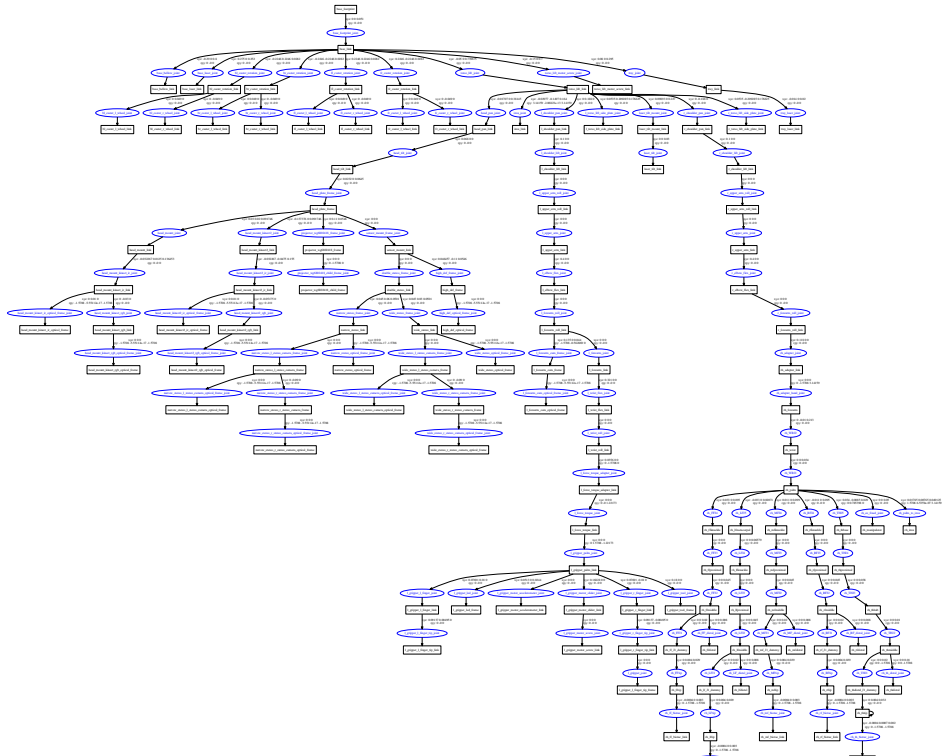
`safety_controller` `soft_lower_limit`, `soft_upper_limit`, `k_position`, `k_velocity`



- ▶ sensor
  - ▶ position and orientation relative to link
  - ▶ sensor properties
    - ▶ update rate
    - ▶ resolution
    - ▶ minimum / maximum angle
- ▶ transmissions
  - ▶ relation of motor to joint motion
- ▶ gazebo
  - ▶ simulation properties
- ▶ model state
  - ▶ description of different robot configurations

## Complex Hierachy

Full URDF hierarchy of the TAMS PR2 with the Shadow Hand.







Introduction

Spatial Description and Transformations

Forward Kinematics

Robot Description

Inverse Kinematics for Manipulators

- Workspace

- Algebraic solvability of manipulator

- Geometrical solvability of manipulator

- Popular inverse kinematics solutions

Velocity of rigid body

Jacobian

Trajectory planning

Trajectory generation

Dynamics





# Outline (cont.)

Inverse Kinematics for Manipulators

Introduction to Robotics

Robot Control

Task-Level Programming and Trajectory Generation

Task-level Programming and Path Planning

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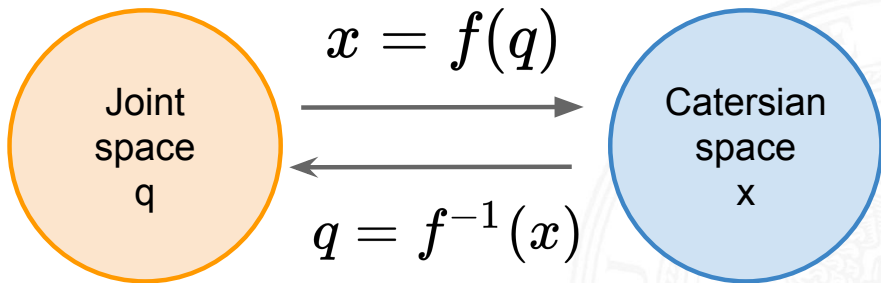
- ▶ **Forward Kinematics:** given robot configurations(joint angles), find position & orientations of the end-effector

## Set of problems

- ▶ In the majority of cases the control of robot manipulators takes place in the *joint space*,
- ▶ The informations about objects are mostly given in the *cartesian space*.



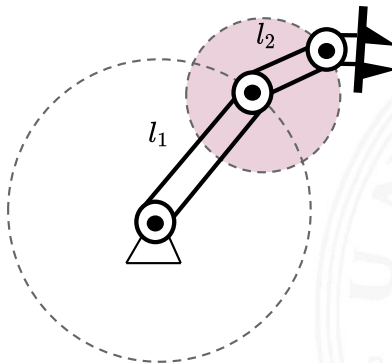
- ▶ **Inverse Kinematics:** give position & orientations of the end-effector, find robot configurations(joint angles)



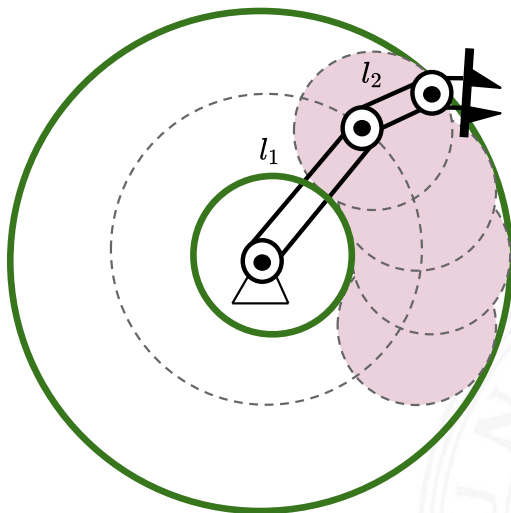
# Existence of solutions: Workspace

Workspace: the volume of space that is reachable for the tool of the manipulator.

- ▶ reachable workspace
- ▶ dexterous workspace

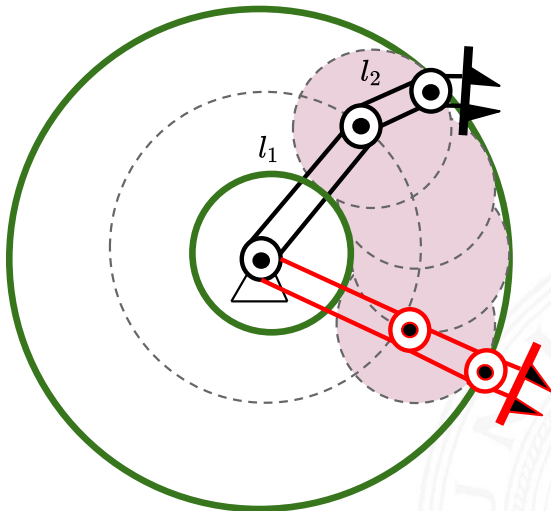


# Existence of solutions: Workspace (cont.)



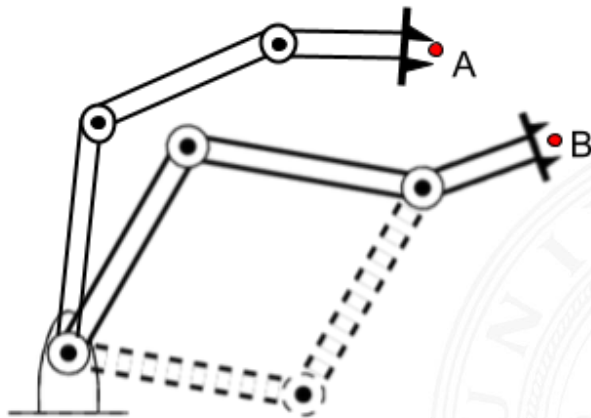
if  $l_1 \neq l_2$ , the reachable workspace becomes a ring of outer radius  $|l_1 + l_2|$ , and inner radius  $|l_1 - l_2|$ .

# Existence of solutions: Workspace (cont.)



Does the workspace change if joint limits are considered?  
For example,  $q_1 \in [0, \pi]$ ,  $q_2 \in [0, \pi]$ .

# Multiple solutions







# The solution using the example of PUMA 560

$$T_6 = T' T'' = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where

$$n_x = C_1[C_{23}(C_4 C_5 C_6 - S_4 S_6) - S_{23} S_5 C_6] - S_1(S_4 C_5 C_6 + C_4 S_6) \quad (2)$$

$$n_y = S_1[C_{23}(C_4 C_5 C_6 - S_4 S_6 - S_{23} S_5 S_6) + C_1(S_4 C_5 C_6 + C_4 S_6)] \quad (3)$$

$$n_z = -S_{23}[C_4 C_5 C_6 - S_4 S_6] - C_{23} S_5 C_6 \quad (4)$$



# The solution using the example of PUMA 560 (cont.)

$$o_x = \dots \quad (5)$$

$$o_y = \dots \quad (6)$$

$$o_z = \dots \quad (7)$$

$$a_x = \dots \quad (8)$$

$$a_y = \dots \quad (9)$$

$$a_z = \dots \quad (10)$$

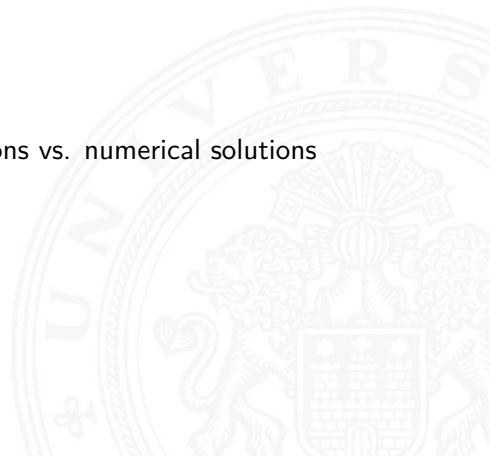
$$p_x = C_1[d_6(C_{23}C_4S_5 + S_{23}C_5) + S_{23}d_4 + a_3C_{23} + a_2C_2] - S_1(d_6S_4S_5 + d_2) \quad (11)$$

$$p_y = S_1[d_6(C_{23}C_4S_5 + S_{23}C_5) + S_{23}d_4 + s_3C_{23} + a_2C_2] + C_1(d_6S_4S_5 + d_2) \quad (12)$$

$$p_z = d_6(C_{23}C_5 - S_{23}C_4S_5) + C_{23}d_4 - a_3S_{23} - a_2S_2 \quad (13)$$



- ▶ Non-linear equations
- ▶ Existence of solutions
- ▶ Multiple solutions
- ▶ Different solution strategy: closed solutions vs. numerical solutions

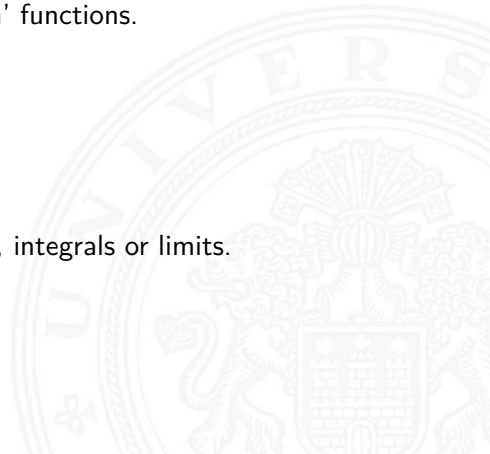




Closed form (analytical):

An expression is said to be a closed-form expression if it can be expressed analytically in terms of a bounded number of certain 'well-known' functions.

- $+ - \times \div$
- nth roots
- exponent and logarithm
- trigonometric and inverse trigonometric functions
- Do not include infinite series, continued fractions, integrals or limits.





Closed form (analytical):

- ▶ algebraic solution
  - + accurate solution by means of equations
  - solution is not geometrically representative
- ▶ geometrical solution
  - + case-by-case analysis of possible robot configurations
  - robot specific

Numerical form:

- ▶ iterative methods
  - + the methods are transferable
  - computationally intensive, for several exceptions the convergence can not be guaranteed



Algebraic Approach manipulates the given equations into a form whose solution is known.

► Method1: **Transcendental equations**

1.  $\sin \theta = a \Rightarrow \theta = A \tan 2(a, \pm\sqrt{1-a^2})$

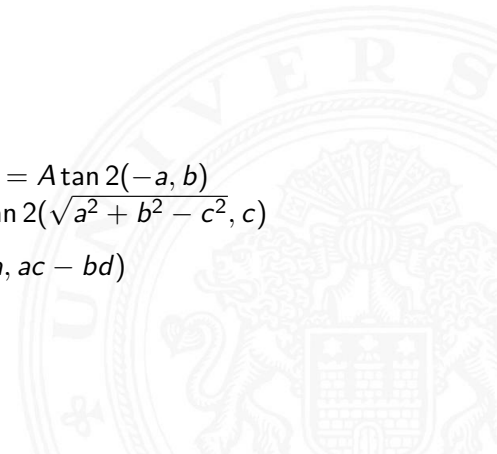
2.  $\cos \theta = b \Rightarrow \theta = \pm A \tan 2(\sqrt{1-b^2}, b)$

3.  $\begin{cases} \sin \theta = a \\ \cos \theta = b \end{cases} \Rightarrow \theta = A \tan 2(a, b)$

4.  $a \cos \theta + b \sin \theta = 0 \Rightarrow \theta = A \tan 2(a, -b)$  or  $\theta = A \tan 2(-a, b)$

5.  $a \cos \theta + b \sin \theta = c \Rightarrow \theta = A \tan 2(b, a) \pm A \tan 2(\sqrt{a^2 + b^2 - c^2}, c)$

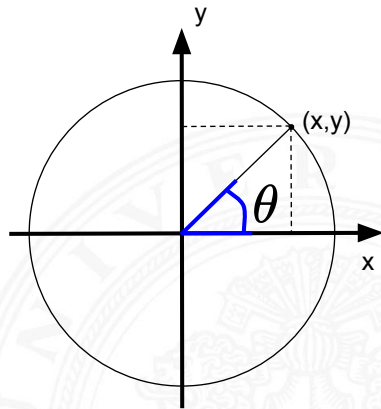
6.  $\begin{cases} a \cos \theta - b \sin \theta = c \\ a \sin \theta + b \cos \theta = d \end{cases} \Rightarrow \theta = A \tan 2(ad - ba, ac - bd)$



# Algebraic solution (cont.)

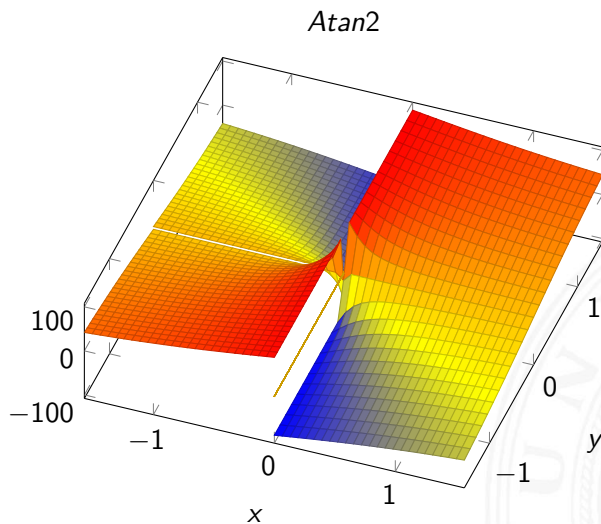
We define the function  $Atan2$  as:

$$\theta = Atan2(y, x) = \begin{cases} Atan(\frac{y}{x}) & \text{for } +x \\ Atan(\frac{y}{x}) + \pi & \text{for } -x, +y \\ Atan(\frac{y}{x}) - \pi & \text{for } -x, -y \\ \frac{\pi}{2} & \text{for } x = 0, +y \\ \frac{-\pi}{2} & \text{for } x = 0, -y \\ NaN & \text{for } x = 0, y = 0 \end{cases}$$



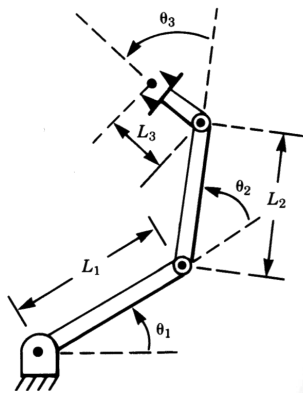


# Algebraic solution (cont.)





# Example: a planar 3 DOF manipulator



Joint	$\alpha_{i-1}$	$a_{i-1}$	$d_i$	$\theta_i$
1	0	0	0	$\theta_1$
2	0	$l_1$	0	$\theta_2$
3	0	$l_2$	0	$\theta_3$

# The algebraical solution for the 3 DOF planar

$$T_6 = {}^0T_3 = \begin{bmatrix} C_{123} & -S_{123} & 0 & l_1 C_1 + l_2 C_{12} \\ S_{123} & C_{123} & 0 & l_1 S_1 + l_2 S_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

with  $C_{ij[k]} = \cos(\theta_i + \theta_j [+ \theta_k])$

Specification for the TCP:  $(x, y, \phi)$ . For such kind of vectors applies:

$${}^0T_3 = \begin{bmatrix} C_\phi & -S_\phi & 0 & x \\ S_\phi & C_\phi & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



# The algebraical solution for the 3 DOF planar (cont.)

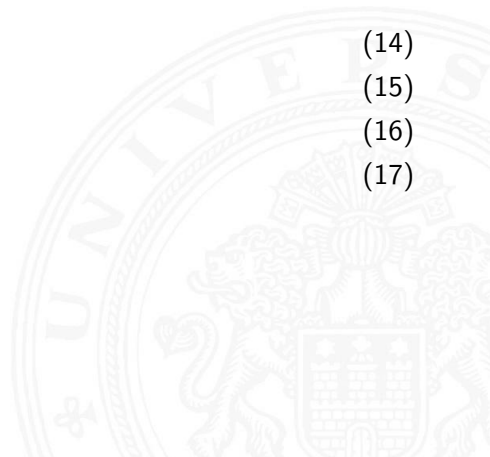
Resultant, four equations can be derived:

$$C_\phi = C_{123} \quad (14)$$

$$S_\phi = S_{123} \quad (15)$$

$$x = l_1 C_1 + l_2 C_{12} \quad (16)$$

$$y = l_1 S_1 + l_2 S_{12} \quad (17)$$





# The algebraical solution for the 3 DOF planar (cont.)

Square and add (20) ( $x = l_1 C_1 + l_2 C_{12}$ ) and (21) ( $y = l_1 S_1 + l_2 S_{12}$ )

$$x^2 + y^2 = l_1^2 + l_2^2 + 2l_1 l_2 C_2$$

using

$$C_{12} = C_1 C_2 - S_1 S_2, S_{12} = C_1 S_2 + S_1 C_2$$

giving

$$C_2 = \frac{x^2 + y^2 - l_1^2 - l_2^2}{2l_1 l_2}$$

for goal in workspace

$$S_2 = \pm \sqrt{1 - C_2^2}$$

solution

$$\theta_2 = \text{atan2}(S_2, C_2)$$



# The algebraical solution for the 3 DOF planar (cont.)

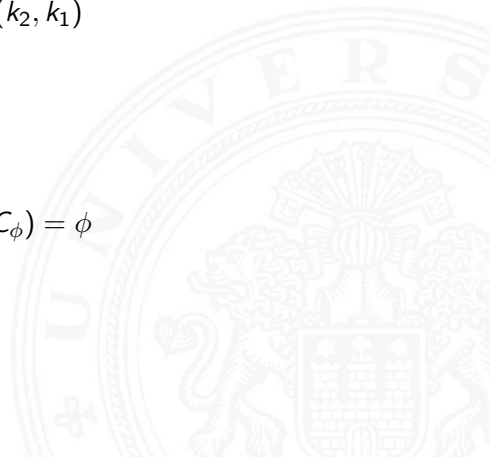
solve (20) ( $x = l_1 C_1 + l_2 C_{12}$ ) and (21) ( $y = l_1 S_1 + l_2 S_{12}$ ) for  $\theta_1$

$$\theta_1 = \text{atan2}(y, x) - \text{atan2}(k_2, k_1)$$

where  $k_1 = l_1 + l_2 C_2$  and  $k_2 = l_2 S_2$ .

solve  $\theta_3$  from (19) ( $C_\phi = C_{123}$ ) and (18) ( $S_\phi = S_{123}$ )

$$\theta_1 + \theta_2 + \theta_3 = \text{atan2}(S_\phi, C_\phi) = \phi$$





The closed solution exists if specific constraints (sufficient constraints) for the arm geometry are satisfied:

**If** 3 sequent axes intersect in a given point  
**or if** 3 sequent axes are parallel to each other

- ▶ manipulators should be designed regarding these constraints
- ▶ most of them are
  - ▶ PUMA 560: axes 4, 5 & 6 intersect in a single point
  - ▶ Mitsubishi PA10, KUKA LWR, PR2
  - ▶ 3-DOF planar (RPC)



## Method2: **Reduction to polynomial**

The following substitutions are used for the polynomial conversion of transcendental equations:

$$u = \tan \frac{\theta}{2}$$

$$\cos \theta = \frac{1 - u^2}{1 + u^2}$$

$$\sin \theta = \frac{2u}{1 + u^2}$$





Example:

The following transcendental equation is given:

$$a \cos \theta + b \sin \theta = c$$

$$\Rightarrow \theta = A \tan 2(b, a) \pm A \tan 2(\sqrt{a^2 + b^2 - c^2}, c)$$

After the polynomial conversion:

$$a(1 - u^2) + 2bu = c(1 + u^2)$$

The solution for  $u$ :

$$u = \frac{b \pm \sqrt{b^2 - a^2 - c^2}}{a + c}$$

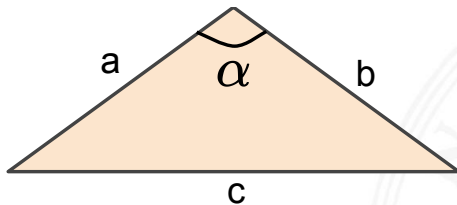
Then:

$$\theta = 2 \tan^{-1} \left( \frac{b \pm \sqrt{b^2 - a^2 - c^2}}{a + c} \right)$$

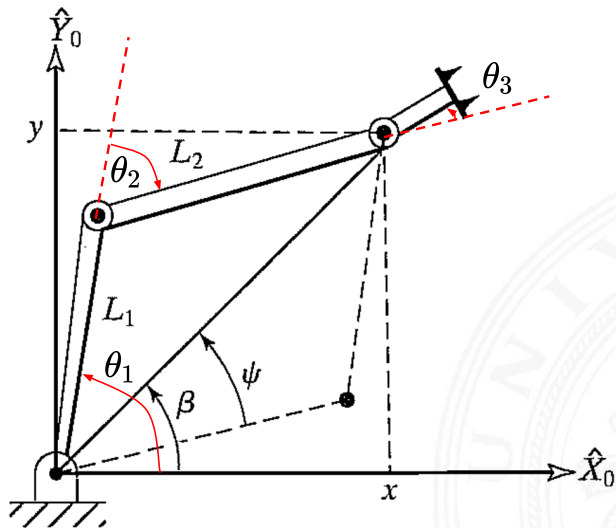




- ▶ Decompose the spatial geometry of the arm into several plane geometry problems
- ▶ Law of cosines:  $c^2 = a^2 + b^2 - 2ab \cos \alpha$



# The geometrical solution for the example 1





Calculate  $\theta_2$  via the law of cosines:

$$x^2 + y^2 = l_1^2 + l_2^2 - 2l_1l_2 \cos(180 + \theta_2)$$

The solution:

$$\theta_2 = \pm \cos^{-1} \frac{x^2 + y^2 - l_1^2 - l_2^2}{2l_1l_2}$$

$$\theta_1 = \beta \pm \psi$$

where:

$$\beta = \text{atan2}_m(y, x), \quad \cos \psi = \frac{x^2 + y^2 - l_1^2 - l_2^2}{2l_1 \sqrt{x^2 + y^2}}$$

For  $\theta_1, \theta_2, \theta_3$  applies:

$$\theta_1 + \theta_2 + \theta_3 = \phi$$



Assume we have derived the forward kinematics as:

$${}^0T_3 = \begin{bmatrix} C_1 C_{23} & -C_1 S_{23} & S_1 & C_1(C_2 l_2 + l_1) \\ S_1 C_{23} & -S_1 S_{23} & -C_1 & S_1(C_2 l_2 + l_1) \\ S_{23} & C_{23} & 0 & S_2 l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

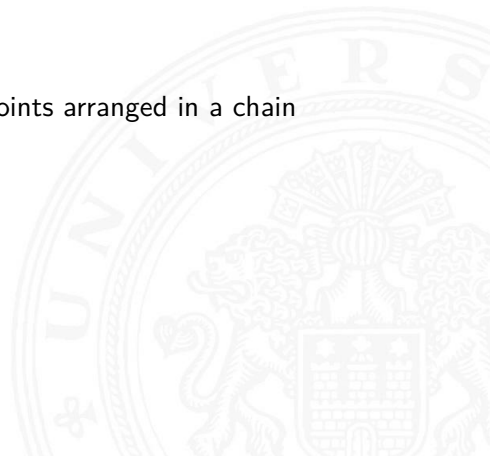
And we know:

$${}^0T_3 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Question: How to solve the inverse kinematics?

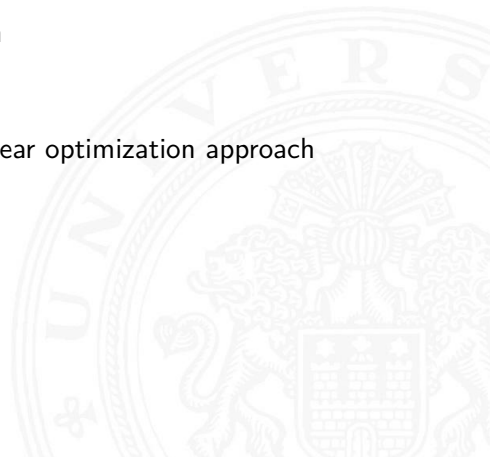


- ▶ Closed-form solutions
- ▶ OpenRAVE
- ▶ faster ( $4 \mu s$ ) but only work with any number of joints arranged in a chain
- ▶ Tutorial: ikfast MoveIt! kinematics\_base plugin



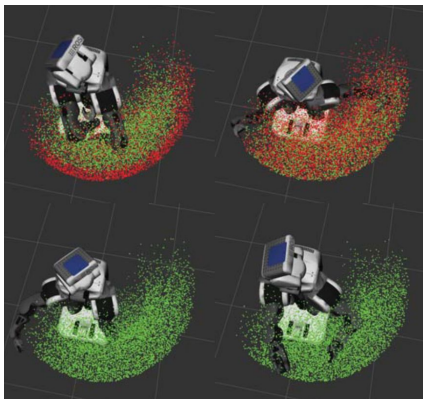


- ▶ TRAC Labs' IK solver
- ▶ Tutorial: `trac_ik` MoveIt! `kinematics_base` plugin
- ▶ two IK implementations:
  - KDL's Newton-based convergence algorithm
  - SQP (Sequential Quadratic Programming) nonlinear optimization approach
- ▶ `trac_ik_python` (RPC)





BioIK 1



KDL

TrackIK

BioIK 2

Download link: [bio\\_ik MoveIt! kinematics\\_base plugin](#)

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<sup>22</sup>Ruppel, P., Hendrich, N., Starke, S. and Zhang, J., 2018, May. Cost functions to specify full-body motion and multi-goal manipulation tasks. In 2018 ICRA (pp. 3152-3159). IEEE.



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