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MIN Faculty
Department of Informatics

Introduction to Robotics

Lecture 6

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Technical Aspects of Multimodal Systems

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Outline

Outline (cont.)

Trajectory

Definition

A trajectory is a time history of

position,

velocity and

acceleration

for each DOF

Describes motion of TCP frame relative to base frame

- abstract from joint configuration

Series of discrete poses (TCP or joint configuration)

- usually fixed temporal intervals
- possibly fixed distances, key frames

Problem

I am at point A and want move to point B.

- How do I get to point B?
- How long does it take me to get to point B?
- Which constraints exist for moving from A to B?

Solution

- generate a possible trajectory
- trajectory planning
- describe intermediate poses (waypoints)

Requirement

The methods for path generation should be applicable for

- calculation of cartesian trajectories for the TCP
- calculation for trajectories in joint space

Primitive solution

Naive approach

Set the pose for the next time step (e.g. 10 ms later) to B.

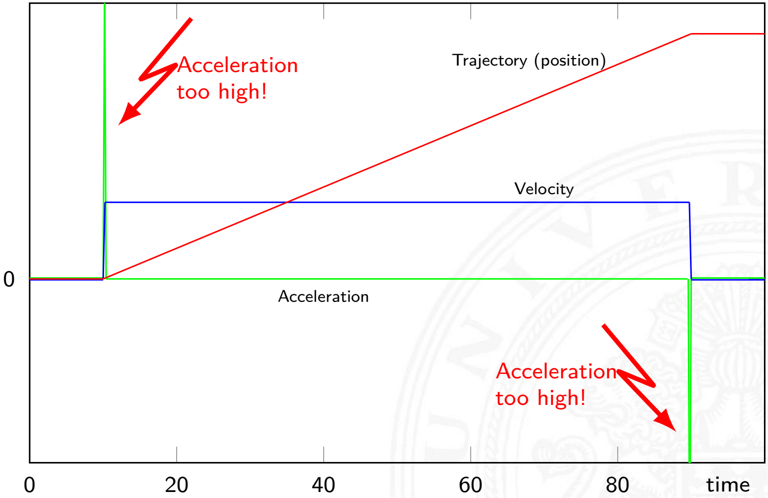
- possible only in simulation
- the moving distance for a manipulator at the next time step may be too large (velocity approaches ∞)

Linear interpolation

Next best approach

- divide distance between A and B to shorter (sub-)distances
- use linear interpolation for these (sub-)distances
- respect the maximum velocity constraint

Linear interpolation – visualization



Linear interpolation – constraints

Problem

The physical constraints are violated

- joint velocity is limited by maximum motor rotation speed
- joint acceleration is limited by maximum motor torque

Implicitly these contraints are valid for motion in cartesian space.

- robot dynamics (joint moments resulting from the robot motion) affect the boundary condition

Solution

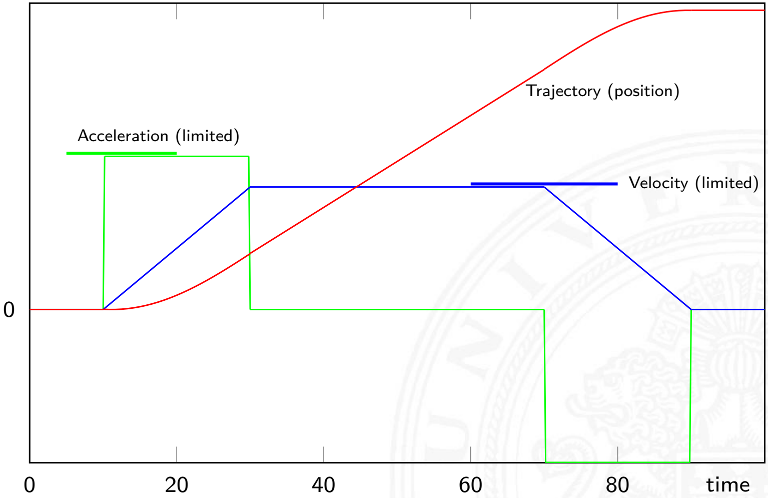
- dynamical trajectory planning
- advanced optimization methods $\rightarrow$ current topic of research

Linear interpolation – improvement

Next best approach

- Limitation of joint velocity and acceleration
- Two different methods
  - trapezoidal interpolation
  - polynomial interpolation

Trapezoidal interpolation – visualization



Trapezoidal interpolation

- consider joint velocity and acceleration contraints
- optimal time usage (move with maximum acceleration and velocity)
- acceleration is not differentiable (the jerk is not continuous)
- start and end velocity equals 0
  - not sensible for concatenating trajectories
  - improved by polynomial interpolation

Trapezoidal interpolation – constraints

Problem

Multidimensional trapezoidal interpolations

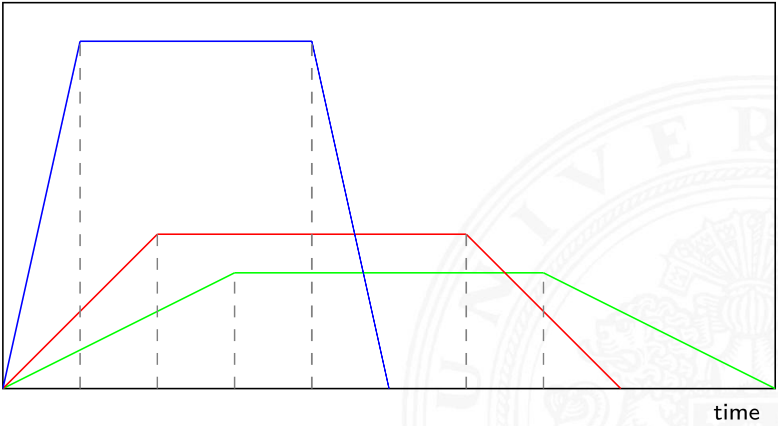
- different run time for joints (or cartesian dimensions)
- multiple velocity and acceleration contraints
- results in various `time switch points`
  - from acceleration to continuous velocity
  - from continuous velocity to deceleration
  - moving along a line in joint/cartesian space is impossible.

Solution

- Normalization to the slowest joint
- Use jerk and arrival time of the slowest joint instead of velocity.

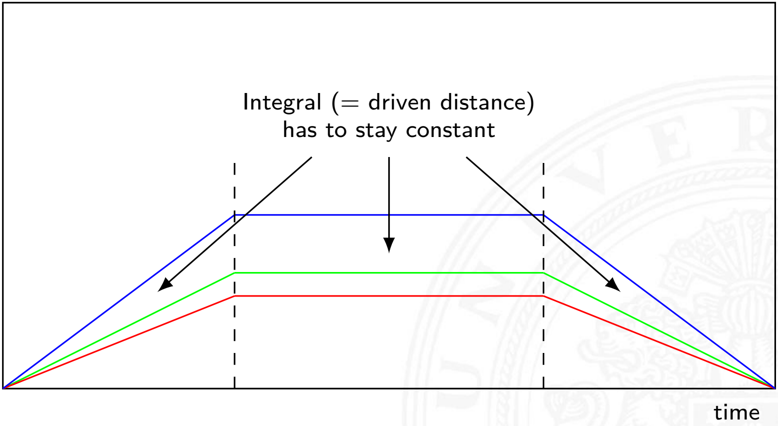
Trapezoidal interpolation – normalization

Normalize to the slowest joint



Trapezoidal interpolation – normalization (cont.)

Normalize to the slowest joint



Polynomial interpolation

- Consider velocity and acceleration boundary conditions
  - calculation of extremum and duration of trajectory
- Acceleration differentiable
  - continous jerk
  - `smooth trajectory`
  - interesting only in the theory – for momentum control
- Start and end velocity may be $\neq 0$
  - sensible for concatenating trajectories

Polynomial interpolation (cont.)

- Usually a polynom with degree of 3 (cubic spline) or 5
- Calculation of coefficient with respect to boundary constraints
  - 3^{rd}-degree polynomial: consider 4 boundary constraints
    - position and velocity; start and goal
  - 5^{th}-degree polynomial: consider 6 boundary constraints
    - position, velocity and acceleration; start and goal

Polynomial interpolation (cont.)

Example 5^{th}-degree

$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5$

Boundary conditions for start ($x = t_0$) and goal ($x = t_d$):

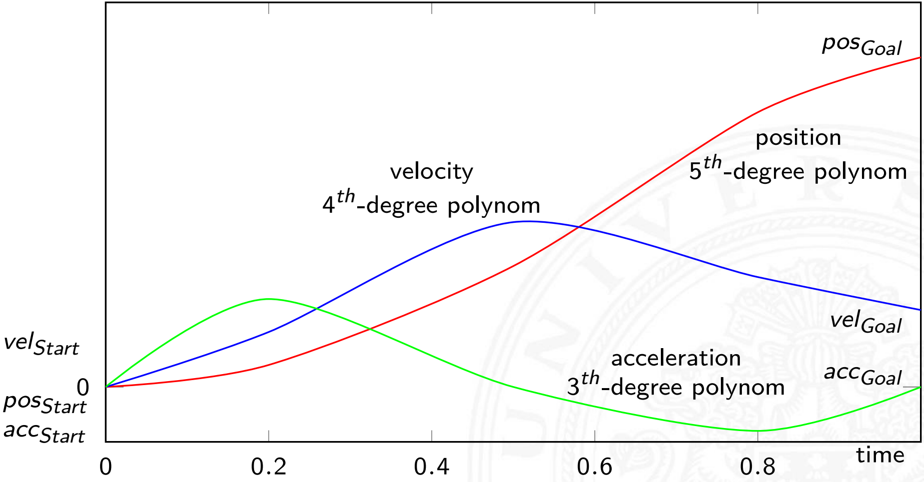
- $f(t_0) = pos_{Start}$, $f(t_d) = pos_{Goal}$
- $f'(t_0) = vel_{Start}$, $f'(t_d) = vel_{Goal}$
- $f''(t_0) = acc_{Start}$, $f''(t_d) = acc_{Goal}$

t: `formal` time from the interval [0;1]

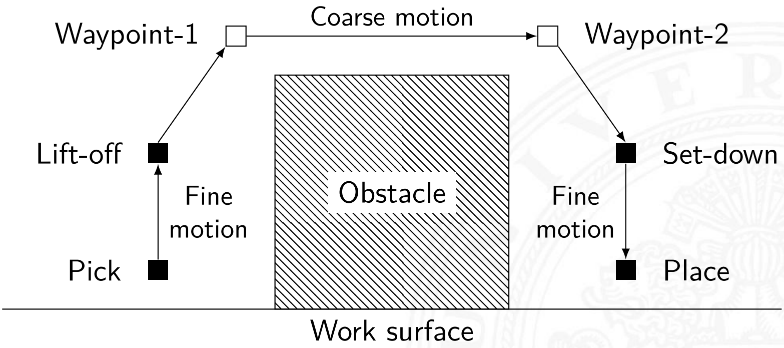
Proper position interpolation from start (A) to goal (B)

$$P(t) = Af(t) + Bf(1-t)$$

Polynomial interpolation (cont.)



Boundary constraints
Pick-and-Place example



Boundary constraints (cont.)

Pick-and-Place example

Pick	$pos_{Start} = object$, $vel_{Start} = 0$, $acc_{Start} = 0$
Lift-off	limited velocity and acceleration
Motion	continuous via waypoints, full velocity and acceleration
Set-down	similar to Lift-off
Place	similar to Pick

Generation of trajectories

Task

- find trajectory for moving the robot from start to goal pose
  - calculate
  - interpolate
  - approximate
- use continous functions of time

Solution:

- Cartesian space
- Joint Space

Generation of trajectories (cont.)

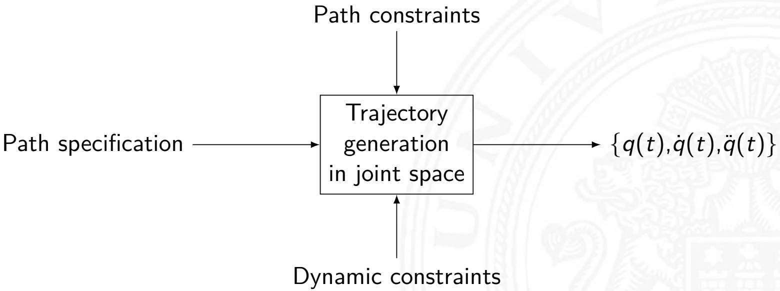
Cartesian space:

- near to the task specification
- advantageous for collision avoidance

Generation of trajectories (cont.)

Joint space:

- no inverse kinematics in joint space required
- the planned trajectory can be immediately applied
- physical joint constraints can be considered



Trajectories in multidimensional space

- Changes in position, velocity and acceleration of all joints are analyzed over a period of time
- Trajectory with n DOF is a parameterized function $q(t)$ with values in its motion region.
- Trajectory $q(t)$ of a robot with n DOF is then a vector of n parameterized functions $q_i(t), i \in \{1 \dots n\}$ with one common parameter t:

$$q(t) = [q_1(t), q_2(t), \dots, q_n(t)]^T$$

Continuity of Trajectories

- A trajectory is C^k-continuous, if all derivatives up to the k-th (including) exist and are continuous.
- A trajectory is called *smooth*, if it is at least C^2-continuous

- $q(x)$ is the trajectory,
- $\dot{q}(x)$ is the velocity,
- $\ddot{q}(x)$ is the acceleration,
- $\dddot{q}(x)$ is the jerk

Remarks on generation of trajectories

- The smoothest curves are generated by infinitly often differentiable functions.
  - e^x
  - $sin(x), \; cos(x)$
  - $log(x)$ (for $x > 0$)
  - ...
- Polynomials are suitable for interpolation
  - Problem: oscillations caused by a degree which is too high
- Piecewise polynomials with specified degree are applicable
  - cubic polynomial
  - splines
  - B-Splines
  - ...

Cubic polynomials between two configurations

- third-degree polynomial $\Rightarrow$ four constraints:

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

- if the start and end velocity is 0 then

$$\theta(0) = \theta_0 \tag{70}$$

$$\theta(t_f) = \theta_f \tag{71}$$

$$\dot{\theta}(0) = 0 \tag{72}$$

$$\dot{\theta}(t_f) = 0 \tag{73}$$

Cubic polynomials between two configurations (cont.)

- The solution

$$
\begin{aligned}
\text{eq. (70)} \quad & a_0 = \theta_0 \\
\text{eq. (72)} \quad & a_1 = 0 \\
& a_2 = \frac{3}{t_f^2}(\theta_f - \theta_0) \\
& a_3 = -\frac{2}{t_f^3}(\theta_f - \theta_0)
\end{aligned}
$$

Cubic polynomials with waypoints and velocities

- Similar to the previous example:
  - positions of waypoints are given (same)
  - velocities of waypoints are different from 0 (different)

$$\theta(0) = \theta_0 \tag{74}$$

$$\theta(t_f) = \theta_f \tag{75}$$

$$\dot{\theta}(0) = \dot{\theta}_0 \tag{76}$$

$$\dot{\theta}(t_f) = \dot{\theta}_f \tag{77}$$

Cubic polynomials with waypoints and velocities (cont.)

- The solution

$$\text{eq. (74)} \quad a_0 = \theta_0$$

$$\text{eq. (76)} \quad a_1 = \dot{\theta}_0$$

$$a_2 = \frac{3}{t_f^2}(\theta_f - \theta_0) - \frac{2}{t_f}\dot{\theta}_0 - \frac{1}{t_f}\dot{\theta}_f$$

$$a_3 = -\frac{2}{t_f^3}(\theta_f - \theta_0) + \frac{1}{t_f^2}(\dot{\theta}_f + \dot{\theta}_0)$$

Velocity calculation at the waypoints

- Manually specify waypoints
  - based on cartesian linear and angle velocity of the tool frame
- Automatic calculation of waypoints in cartesian or joint space
  - based on heuristics
- Automatic determination of the parameters
  - based on continous acceleration at the waypoints

Factors for time optimal motion – Arc Length

If the curve in the n-dimensional K space is given by

$$\mathbf{q}(t) = [q^1(t), q^2(t), \ldots, q^n(t)]^T$$

then the **arc length** can be defined as follows:

$$s = \int_0^t \|\dot{\mathbf{q}}(t)\|_2 \, dt$$

where $\|\dot{\mathbf{q}}(t)\|_2$ is the euclidean norm of vector $d\mathbf{q}(t)/dt$ and is labeled as a flow velocity along the curve.

$$\|\mathbf{x}\|_2 := \sqrt{x_1^2 + \cdots + x_n^2}$$

Factors for time optimal motion – Arc Length (cont.)

With the following two points given
$\mathbf{p}_0 = \mathbf{q}(t_s)$ und $\mathbf{p}_1 = \mathbf{q}(t_f)$,
the arc length L between $\mathbf{p}_0$ and $\mathbf{p}_1$ is the integral:

$$L = \int_{\mathbf{p}_1}^{\mathbf{p}_0} ds = \int_{t_s}^{t_f} \|\dot{\mathbf{q}}(t)\|_2 \, dt$$

> *"The trajectory parameters should be calculated in the way that the arc length L under the given constraints has the shortest possible value."*

Factors for time optimal motion – Arc Length (cont.)

- trajectory of circle

$$q(t) = c(t) = [r\cos(t), r\sin(t)]^T$$

- arc length L of circle (circumference)

$$L = \int_0^{2\pi} \|\dot{\mathbf{c}}(t)\|_2 \, dt \tag{78}$$

$$= \int_0^{2\pi} \left\| [-r\sin(t), r\cos(t)]^T \right\|_2 dt \tag{79}$$

$$= \int_0^{2\pi} \sqrt{r^2(\sin^2(t) + \cos^2(t))} dt \tag{80}$$

$$= \int_0^{2\pi} r dt \tag{81}$$

$$L = 2\pi r \tag{82}$$

Factors for time optimal motion – Curvature

Curvature

Defines the `sharpness` of a curve. A straight line has zero curvature. Curvature of large circles is smaller than of small circles.

At first the *unit vector* of a curve $\mathbf{q}(t)$ can be defined as

$$\mathbf{U} = \frac{d\mathbf{q}(t)}{ds} = \frac{d\mathbf{q}(t)/dt}{ds/dt} = \frac{\dot{\mathbf{q}}(t)}{|\dot{\mathbf{q}}(t)|}$$

If s is the parameter of the *arc length* and $\mathbf{U}$ as the *unit vector* is given, the **curvature** of curve $\mathbf{q}(t)$ can be defined as

$$\kappa(s) = \left| \frac{d\mathbf{U}}{ds} \right|$$

Factors for time optimal motion – Curvature (cont.)

$$\text{with} \quad \kappa(s) = \left|\frac{d\mathbf{U}}{ds}\right| \quad \rightarrow \text{curvature}$$

If the parameter t, the first derivative $\dot{\mathbf{q}} = {}^{d\mathbf{q}(t)}/_{dt}$ and the second derivative $\ddot{\mathbf{q}} = {}^{d\dot{\mathbf{q}}(t)}/_{dt}$ for the curve $\mathbf{q}(t)$ are given, then the *curvature* can be calculated from the following representation

$$\kappa(t) = \frac{|\dot{\mathbf{q}} \times \ddot{\mathbf{q}}|}{|\dot{\mathbf{q}}^3|} = \frac{\left(\dot{\mathbf{q}}^2 \cdot \ddot{\mathbf{q}}^2 - (\dot{\mathbf{q}} \cdot \ddot{\mathbf{q}})^2\right)^{1/2}}{|\dot{\mathbf{q}}|^3}$$

where $\dot{\mathbf{q}} \times \ddot{\mathbf{q}}$ is the cross product and $\dot{\mathbf{q}} \cdot \ddot{\mathbf{q}}$ is the dot product

http://web.cs.iastate.edu/~cs577/handouts/curvature.pdf with $q(t) = c(t) = [r\cos(t), r\sin(t)]^T \rightarrow$ trajectory of a circle

$$\begin{aligned}
\dot{c}(t) &= [-r\sin(t), r\cos(t)]^T \\
\ddot{c}(t) &= [-r\cos(t), -r\sin(t)]^T \\
\dot{c}^2(t) &= r^2\sin^2(t) + r^2\cos^2(t) = r^2 \\
\ddot{c}^2(t) &= r^2\cos^2(t) + r^2\sin^2(t) = r^2 \\
\dot{c}(t)\cdot\ddot{c}(t) &= r^2\sin(t)\cos(t) - r^2\cos(t)\sin(t) = 0
\end{aligned}$$

Curvature of a circle

$$\kappa(t) = \frac{\left(\dot{\mathbf{c}}^2 \cdot \ddot{\mathbf{c}}^2 - (\dot{\mathbf{c}} \cdot \ddot{\mathbf{c}})^2\right)^{1/2}}{|\dot{\mathbf{c}}|^3} = \frac{\sqrt{r^4}}{r^3} = \frac{1}{r}$$

Factors for time optimal motion – Bending Energy

The **bending energy** of a smooth curve $\mathbf{q}(t)$ over the interval $t \in [0, T]$ is defined as

$$E = \int_0^L \kappa(s)^2 ds = \int_0^T \kappa(t)^2 |\dot{\mathbf{q}}(t)| dt$$

where $\kappa(t)$ is the curvature of $\mathbf{q}(t)$.

> *"The bending energy E of a trajectory should be as small as possible under consideration of the arc length."*

Factors for time optimal motion – Motion Time

If a motion consists of n successive segments

$$q_j, j \in \{1 \dots n\}$$

then

$$u_j = t_{j+1} - t_j$$

is the required time for the motion in the segment $\mathbf{q}_j$. The total motion time is

$$T = \sum_{j=1}^{n-1} u_j$$

Dynamical constraints for all joints

The borders for the minimum motion time T_{min} for the trajectory $\mathbf{q}_j^i(t)$ are defined over dynamical parameters of all joints.

For joint $i \in \{1 \ldots n\}$ of trajectory part $j \in \{1 \ldots m\}$ this kind of constraint can be described as follows

$$|\dot{q}_j^i(t)| \leq \dot{q}_{max}^i \tag{83}$$

$$|\ddot{q}_j^i(t)| \leq \ddot{q}_{max}^i \tag{84}$$

$$|m_j^i(t)| \leq m_{max}^i \tag{85}$$

- m^i is the torque (moment of force) for the joint i and can be calculated from the dynamical equation (motion equation).
- $\dot{q}_{max}^i$, $\ddot{q}_{max}^i$ and m_{max}^i represent the important parameters of the dynamical capacity of the robot.

Difficulties for cartesian space trajectory generation

- Waypoints cannot be realized
  - workspace boundaries, object collision, self-collision
- Velocities in the vicinity of singular configurations are too high
- Start and end configurations can be achieved, but there are different solutions
  - ambiguous solutions

Motion along a line $< \mathbf{w}_0, \mathbf{w}_1 >$

- The following algorithm should create the smallest set of waypoints in the joint space under a predefined deviation $\epsilon > 0$.
- Therefore the deviation between the trajectory $\mathbf{q}(t)$ and the given line $< \mathbf{w}_0, \mathbf{w}_1 >$ must be smaller than ϵ.

Algorithm(Bounded_Deviation)

1. Calculation of possible configurations $\mathbf{q}_0, \mathbf{q}_1$ from $\mathbf{w}_0, \mathbf{w}_1$ with the help of the inverse kinematics.
2. Calculation of the center in joint space:

$$\mathbf{q}_m = \frac{\mathbf{q}_0 + \mathbf{q}_1}{2}$$

Motion along a line $< \mathbf{w}_0, \mathbf{w}_1 >$ (cont.)

3. Calculation of the corresponding point of $\mathbf{q}_m$ in the workspace with usage of direct kinematics:

$$\mathbf{w}_m = W(\mathbf{q}_m)$$

4. Calculation of the center in the workspace:

$$\mathbf{w}_M = \frac{\mathbf{w}_0 + \mathbf{w}_1}{2}$$

5. If the deviation $||\mathbf{w_m} - \mathbf{w_M}|| \geq \epsilon$, then cancel; else add the $\mathbf{w}_M$ as node point between $\mathbf{w}_0$ and $\mathbf{w}_1$.
6. Recursive application of the algorithm for two new segments $(\mathbf{w}_0, \mathbf{w}_M)$ und $(\mathbf{w}_M, \mathbf{w}_1)$.

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