Introduction to Robotics

Lecture 5

Lasse Einig, Jianwei Zhang
[einig, zhang]@informatik.uni-hamburg.de

University of Hamburg
Faculty of Mathematics, Informatics and Natural Sciences
Department of Informatics
Technical Aspects of Multimodal Systems

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Outline

Outline (cont.)

Jacobian of a Manipulator

Definition

- A Jacobian-matrix is a multidimensional representation of partial derivatives.
- The Jacobian of a manipulator links the joint velocities with the cartesian velocity of the TCP.
- The Jacobian matrix depends on the current state of the robot joints.

Jacobian of a Manipulator (cont.)

- Consider an n-link manipulator with joint variables $q_1, q_2, ..q_n$.
- Define $q = [q_1, q_2, ..q_n]^T$
- Let the transformation from base to end-effector frame be:

$$T = \begin{bmatrix} R_n^0(q) & o(q) \\ 0 & 1 \end{bmatrix} \tag{52}$$

- We define ω_n^0 to be the angular velocity of the end-effector
- The linear velocity of the end-effector is v_n^0
- The **Jacobian** matrix consists of two components, that solve the following equations:

$$v_n^0 = J_v \dot{q} \qquad \text{and} \qquad \omega_n^0 = J_w \dot{q}$$

Jacobian of a Manipulator (cont.)

The manipulator Jacobian

$$J := \begin{bmatrix} J_v \\ J_w \end{bmatrix}$$

We define the body velocity of the endeffector:

$$\xi := \begin{bmatrix} v_n^0 \\ \omega_n^0 \end{bmatrix} := \begin{bmatrix} {}^T d_x \\ {}^T d_y \\ {}^T d_z \\ {}^T \delta_x \\ {}^T \delta_y \\ {}^T \delta_z \end{bmatrix} \qquad \xi = J\dot{q}$$

Angular Velocity Jacobian

Revolute joints

If the i^{th} joint is revolute, the axis of rotation is given by z_{i-1}.

Let $\omega_{i-1,i}^{i-1}$ represent the angular velocity of the link i w.r.t. the frame $i-1$.

Then, we have: $\omega_{i-1,i}^{i-1} = \dot{q}_i z_{i-1}^{i-1}$

Prismatic joints

If the i^{th} joint is prismatic, the motion of frame i relative to frame $i-1$ is a translation.

Then, we have: $\omega_{i-1,i}^{i-1} = 0$

Angular Velocity Jacobian (cont.)

Overall angular velocity:

$$\omega_{0,n}^{0} = \omega_{0,1}^{0} + R_{1}^{0}\omega_{1,2}^{1} + \ldots + R_{n-1}^{0}\omega_{n-1,n}^{n-1} \tag{53}$$

We get:

$$\omega_{0,n}^{0} = p_1\dot{q}_1 z_0^0 + p_2\dot{q}_2 R_1^0 z_1^1 + \ldots + p_n\dot{q}_n R_{n-1}^0 z_{n-1}^{n-1} \tag{54}$$

$$= p_1\dot{q}_1 z_0^0 + p_2\dot{q}_2 z_1^0 + \ldots + p_n\dot{q}_n z_{n-1}^0 \tag{55}$$

where:

$$p_i = \begin{cases} 0 & \text{if i is prismatic} \\ 1 & \text{if i is revolute} \end{cases} \tag{56}$$

Angular Velocity Jacobian (cont.)

The complete Jacobian

$$\begin{bmatrix} v_n^0 \\ \omega_n^0 \end{bmatrix} = \begin{bmatrix} J_v \\ J_w \end{bmatrix} \dot{q} \tag{57}$$

The Angular Velocity Jacobian

$$J_w = [p_1 z_0^0 \quad p_2 z_1^0 \quad \dots \quad p_n z_{n-1}^0] \tag{58}$$

(Hint: J_w is a $3xn$ matrix; due to matrix multiplication rules the representation is equal to those on the last slide.)

Linear Velocity Jacobian

The linear velocity of the end effector is: $\dot{o}_n^0$

By the chain rule of differentiation:

$$\dot{o}_n^0 = \frac{\delta o_n^0}{\delta q_1}\dot{q}_1 + \frac{\delta o_n^0}{\delta q_2}\dot{q}_2 + \dots + \frac{\delta o_n^0}{\delta q_n}\dot{q}_n \tag{59}$$

therefore the linear part of the Jacobian is:

$$J_v = \begin{matrix} \frac{\delta o_n^0}{\delta q_1} & \frac{\delta o_n^0}{\delta q_2} & \dots & \frac{\delta o_n^0}{\delta q_n} \end{matrix} \tag{60}$$

Linear Velocity Jacobian – Prismatic

Every prismatic joint influences the velocity of the endeffector depending on:

- the current linear velocity of the joint ($\dot{d}_i$)
- the current orientation of the z-axis of the joint (z_{i-1})
  - depending on q

$$\dot{o}_n^0 = \dot{d}_i z_{i-1} \tag{61}$$

Therefore:

$$J_{v_i} = \frac{\delta o_n^0}{\delta q_n} = z_{i-1} \tag{62}$$

Linear Velocity Jacobian – Revolute

Every revolute joint influences the velocity of the end-effector depending on:

- the current angular velocity of the joint ($\dot{q}_i$)
- the current orientation of the z-axis of the joint (z_{i-1})
- the current vector from the joint origin o_{i-1} to the end-effector
  - the two latter depending on q

The linear velocity of the end-effector is of form:

$$\omega \times r$$

with $\omega = \dot{q}_i z_{i-1}$ and $r = o_n^0 - o_{i-1}^0$

Therefore:

$$J_{v_i} = \frac{\delta o_n^0}{\delta q_n} = z_{i-1} \times (o_n^0 - o_{i-1}^0) \qquad (63)$$

Computing the final Jacobian

$$J := \begin{bmatrix} J_v \\ J_w \end{bmatrix}$$

$$J_v = \begin{bmatrix} J_{v_1} & J_{v_2} & J_{v_n} \end{bmatrix} \quad \text{with} \tag{64}$$

$$J_{v_i} = \begin{cases} z_{i-1} & \text{if i is prismatic} \\ z_{i-1} \times (o_n^0 - o_{i-1}^0) & \text{if i is revolute} \end{cases} \tag{65}$$

and

$$J_w = \begin{bmatrix} J_{w_1} & J_{w_2} & J_{w_n} \end{bmatrix} \quad \text{with} \tag{66}$$

$$J_{w_i} = \begin{cases} 0 & \text{if i is prismatic} \\ z_{i-1} & \text{if i is revolute} \end{cases} \tag{67}$$

Computing the final Jacobian (cont.)

Target

Compute z_i and o_i.

- z_i is equal to the first three elements of the 3rd column of matrix ${}^0T_i$
- o_i is equal to the first three elements of the 4th column of matrix ${}^0T_i$

${}^0T_i$ has to be computed for every joint.

Jacobian of a Manipulator – DOF

Consider a Manipulator with 6 DOFs:

$$T_6 = A_1 A_2 A_3 A_4 A_5 A_6$$

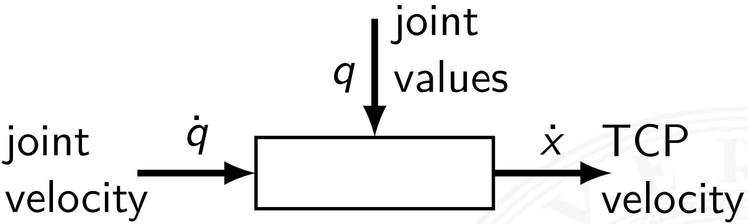
the Jacobian is:

$$\begin{bmatrix} {}^{T_6}d_x \\ {}^{T_6}d_y \\ {}^{T_6}d_z \\ {}^{T_6}\delta_x \\ {}^{T_6}\delta_y \\ {}^{T_6}\delta_z \end{bmatrix} = J_{6\times 6} \begin{bmatrix} dq_1 \\ dq_2 \\ dq_3 \\ dq_4 \\ dq_5 \\ dq_6 \end{bmatrix}$$

$$\dot{\mathbf{x}} = J(\mathbf{q}) \quad \dot{\mathbf{q}}$$

In case of a 6-DOF manipulator, we get a 6×6 matrix.

Inverse Jacobian



Question

Is the Jacobian invertible?

If it is, then: $\dot{\mathbf{q}} = J^{-1}(\mathbf{q})\dot{\mathbf{x}}$

$\Longrightarrow$ to move the the end-effector of the robot in Cartesian Space with a certain velocity.

Singular Configurations

For most manipulators there exist values of $\mathbf{q}$ where the Jacobian gets singular.

Singularity

$$\det J = 0 \Longrightarrow J \text{ is not invertible}$$

Such configurations are called singularities of the manipulator.
Two Main types of Singularities:

- Workspace boundary singularities
- Workspace internal singularities

Singular Configurations – Workarounds

- generally only for 6-DOF manipulators the Jacobian is invertible
- there are workarounds for other types of manipulators

$n < 6$ manually restrict the DOF of the end-effector
$\Longrightarrow$ square Jacobian matrix.
Example:

$$\begin{bmatrix} {}^{T_6}d_x \\ {}^{T_6}d_y \end{bmatrix} = J_{2\times 2} \begin{bmatrix} dq_1 \\ dq_2 \end{bmatrix}$$

for a 2-joint planar manipulator

$n > 6$ use the `pseudoinverse of J`

$$A^+ = (A^T \cdot A)^{-1} \cdot A^T, \text{ linear independent colums} \tag{68}$$

$$A^+ = A^T \cdot (A^T \cdot A)^{-1}, \text{ linear independent rows} \tag{69}$$

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