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# Motion Planning

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Department of Informatics

**Technical Aspects of Multimodal Systems**

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# Outline

Motivation

Kinematics

Potential Functions

Discrete Planning

Conclusion and Outlook

1. Motivation
2. Kinematics
3. Potential Functions
4. Discrete Planning
5. Conclusion and Outlook





*Motion planning [...] is a term used in robotics for the process of breaking down a desired movement task into discrete motions that satisfy movement constraints and possibly optimize some aspect of the movement.*

[https://en.wikipedia.org/wiki/Motion\\_planning](https://en.wikipedia.org/wiki/Motion_planning)





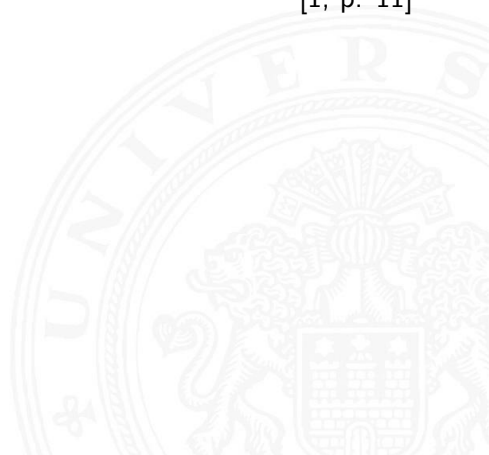
- ▶ Robotic Manipulators
  - ▶ Robotic arms
  - ▶ CNC machines
- ▶ Path planning and execution
  - ▶ Mobile robots
  - ▶ Autonomous cars
- ▶ Computer animation





*Kinematics pertains to the motion of bodies in a robotic mechanism without regard to the forces/torques that cause the motion.*

[1, p. 11]



# Cartesian Coordinates

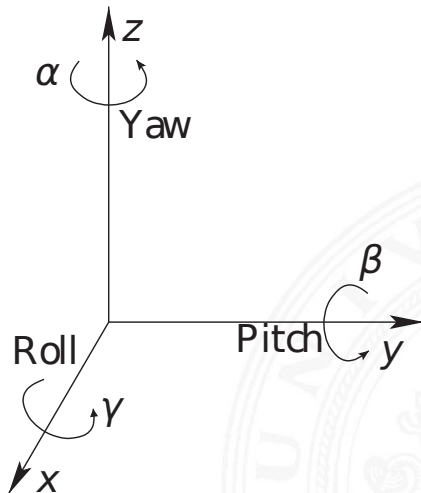
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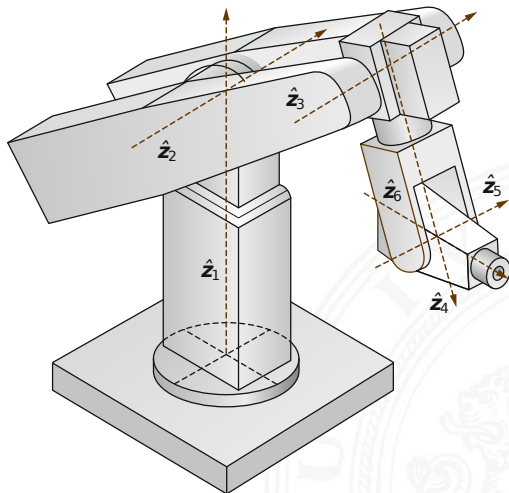
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[2, p. 80]



[1, p. 26]



# Configuration Space

- ▶  $n$  dimensional space where  $n$  is the number degrees of freedom in the robot
- ▶ limited by joint limits
- ▶ transforming from configuration space to Cartesian space:  
Forward Kinematics
- ▶ transforming from Cartesian space to configuration space:  
Inverse Kinematics







# Configuration Space

- ▶  $n$  dimensional space where  $n$  is the number degrees of freedom in the robot
- ▶ limited by joint limits
- ▶ transforming from configuration space to Cartesian space:  
Forward Kinematics
- ▶ transforming from Cartesian space to configuration space:  
Inverse Kinematics
  
- ▶ live demo <http://demonstrations.wolfram.com/RobotMotionWithObstacles/>

# Configuration Space

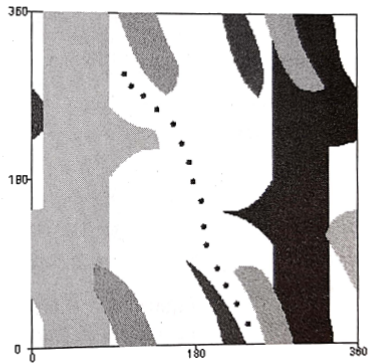
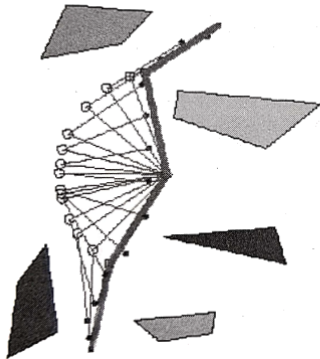
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[3, p. 45]

# Configuration Space

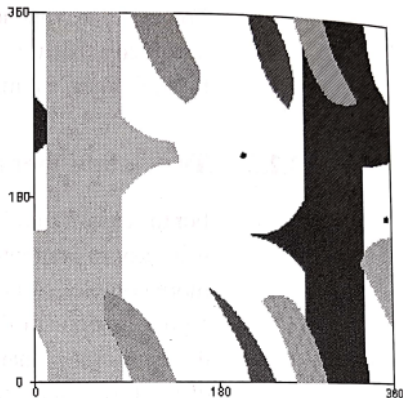
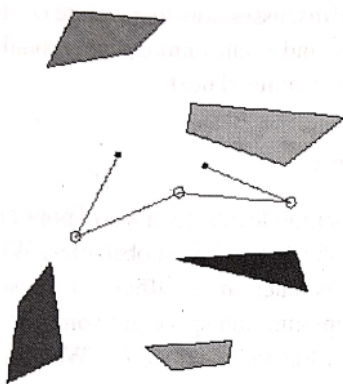
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[3, p. 46]

# Configuration Space

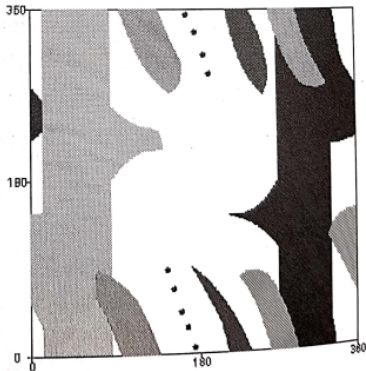
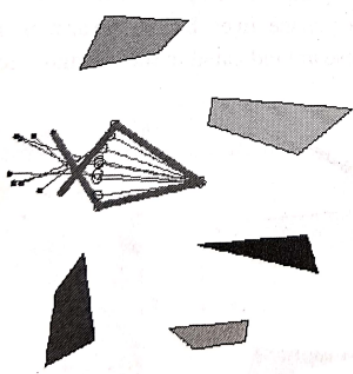
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[3, p. 46]

# Forward kinematics

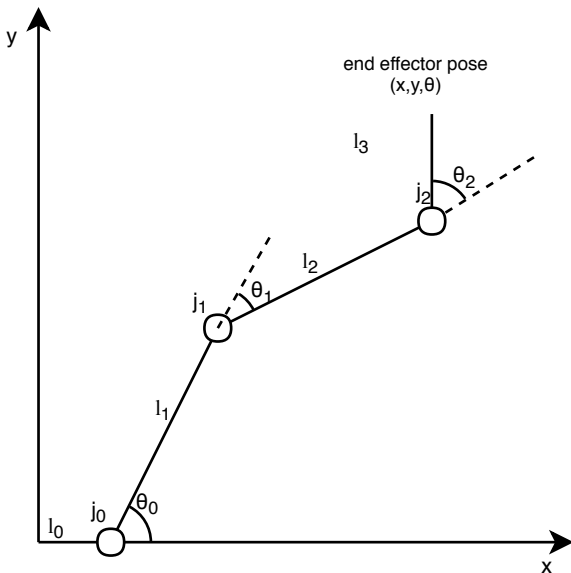
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# Forward kinematics

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- ▶ can be calculated using transformation matrices (geometry)
- ▶ unambiguous
- ▶ fast
- ▶ DH-Parameters [4]
- ▶ URDF<sup>1</sup> + robot\_state\_publisher<sup>2</sup> + tf/tf2<sup>3</sup>

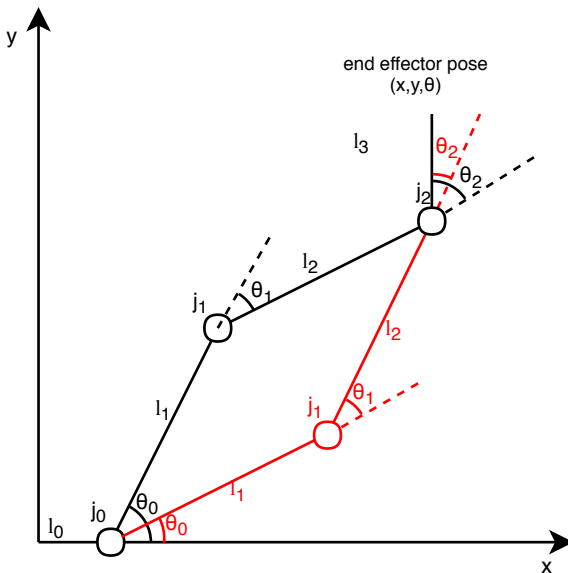
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<sup>1</sup><http://wiki.ros.org/urdf/>

<sup>2</sup>[http://wiki.ros.org/robot\\_state\\_publisher](http://wiki.ros.org/robot_state_publisher)

<sup>3</sup><http://wiki.ros.org/tf> | <http://wiki.ros.org/tf2>

# Inverse kinematics





# Inverse Kinematics

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- ▶ not necessarily unambiguous
- ▶ much harder than Forward Kinematics
- ▶ analytic robot specific solution
- ▶ generic numeric solution







# Inverse Kinematics

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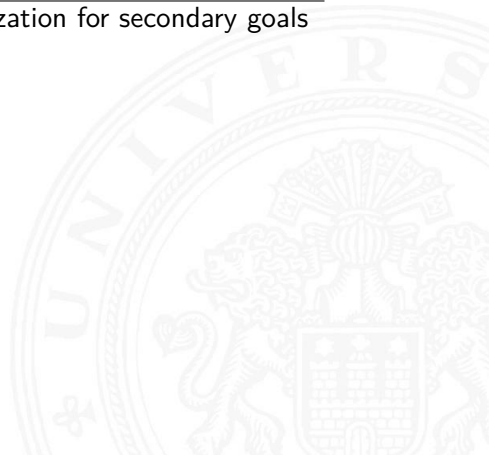
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Analytic	Numeric
fast	comparably slow
robot specific	generic
guarantees correctness	does not guarantee correctness
	optimization for secondary goals





# Examples of Inverse Kinematics engines

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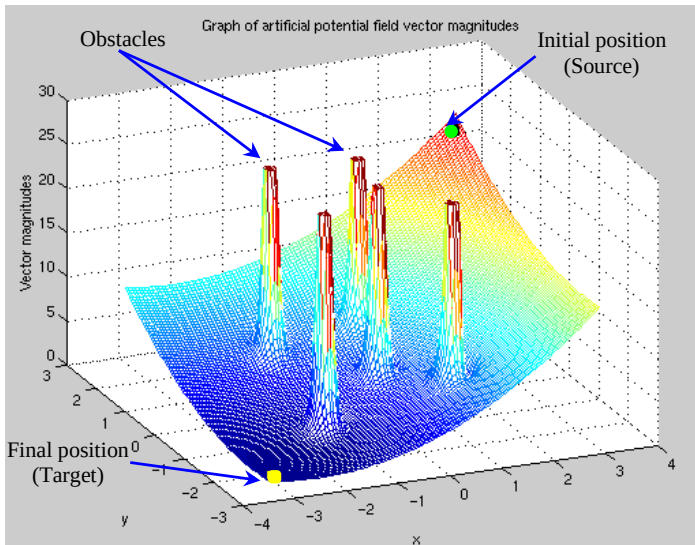
Discrete Planning

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- ▶ OpenRave [5] (analytic)
- ▶ TracK [6] (numeric)
- ▶ BiolK [7] (numeric)



# Potential Functions





# Potential Functions

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- ▶ attractive potential field for goal
- ▶ repulsive potential field for obstacles





# Gradient Descent

**Data:** A means to compute the gradient  $\nabla U(q)$  at a point  $q$

**Result:** A sequence of points  $q(0), q(1), \dots, q(n)$

$q(0) = q_{start};$

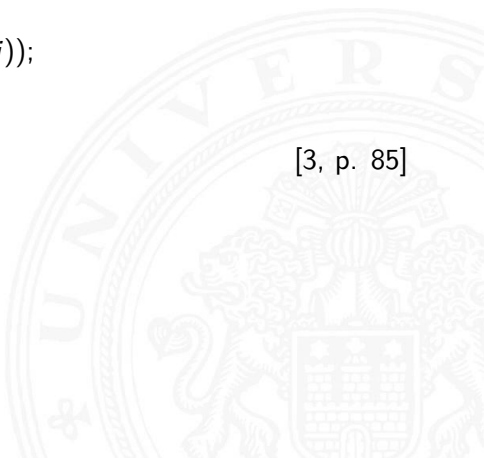
$i = 0;$

**while**  $\nabla U(q(i)) \neq 0$  **do**

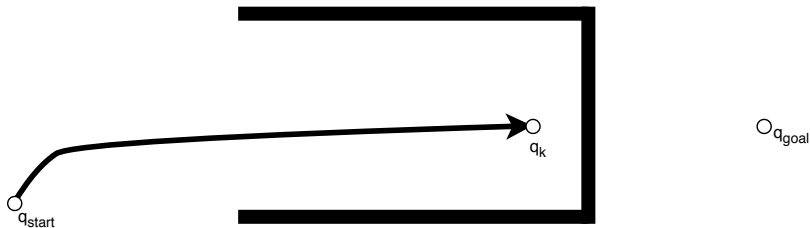
$q(i + 1) = q(i) + \alpha(i)\nabla U(q(i));$   
     $i = i + 1$

**end**

[3, p. 85]

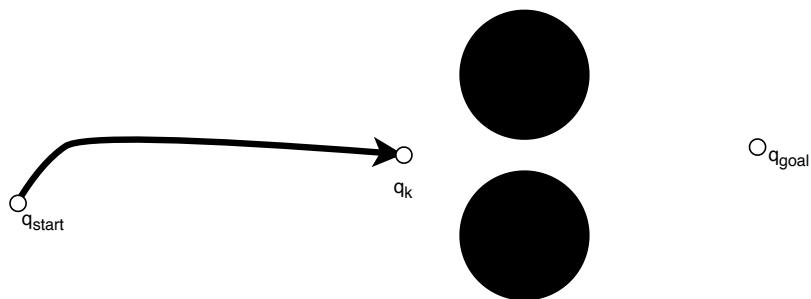


Problem: local minima



inspired by [3, p. 85]

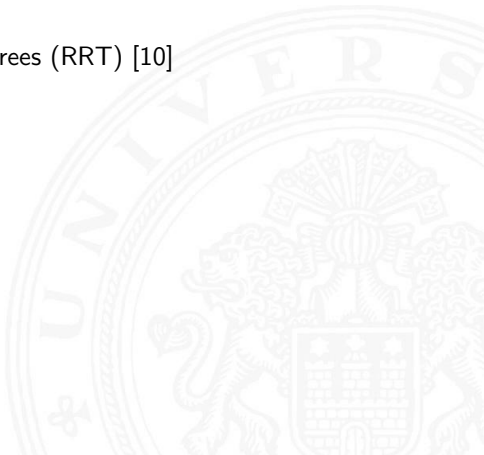
Problem: local minima



inspired by [3, p. 86]



- ▶ discretization of configuration space
- ▶ collision check for each explored state
- ▶ variety of graph creation algorithms
  - ▶ evenly spaced grid
  - ▶ Probabilistic Roadmaps [9]
  - ▶ Rapidly-Exploring Random Trees (RRT) [10]
  - ▶ RRT-Connect [11]







**Data:** A means to check for collision of a configuration  $c$

**Result:** Graph in the configuration space

**repeat**  $n$  **times**

    generate random configuration  $c$ ;

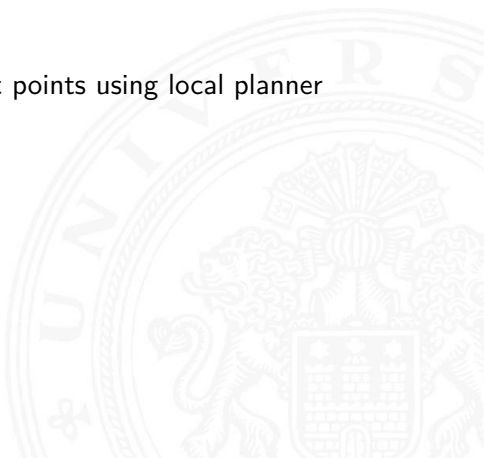
**if**  $c$  *is collision free* **then**

        find  $k$  closest points ;

        try to connect  $c$  to closest points using local planner

**end**

**end**



**Data:** A means to check for collision of a configuration  $c$

**Result:** Graph in the configuration space

**repeat**  $n$  **times**

    generate random configuration  $c$ ;

**if**  $c$  *is collision free* **then**

        find  $k$  closest points ;

        try to connect  $c$  to closest points using local planner

**end**

**end**

local planner can be achieved using a set of interpolated vertices between the points to be connected

# Probabilistic Roadmaps

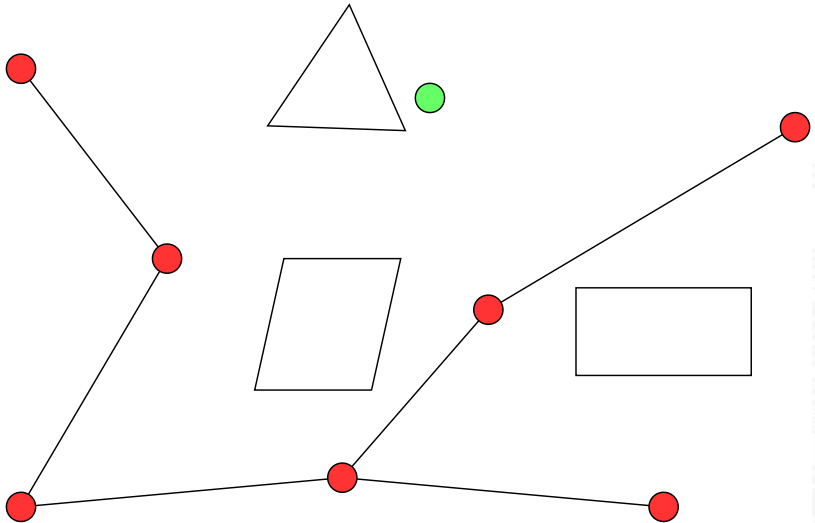
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# Probabilistic Roadmaps

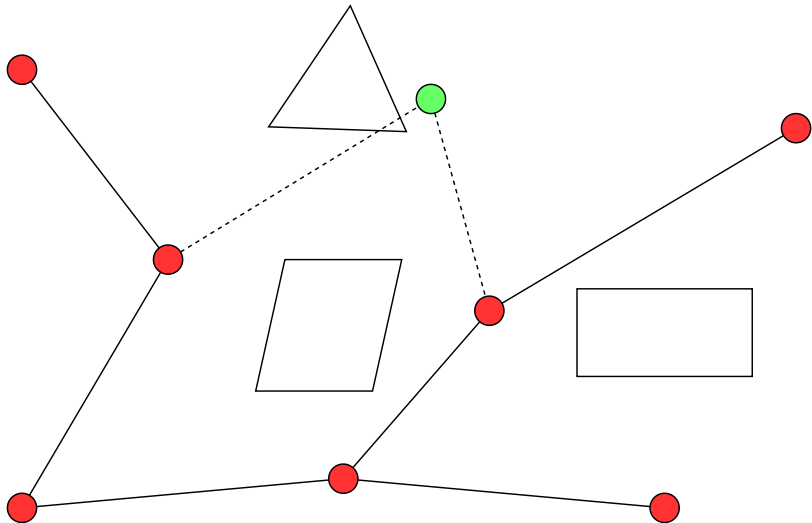
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# Probabilistic Roadmaps

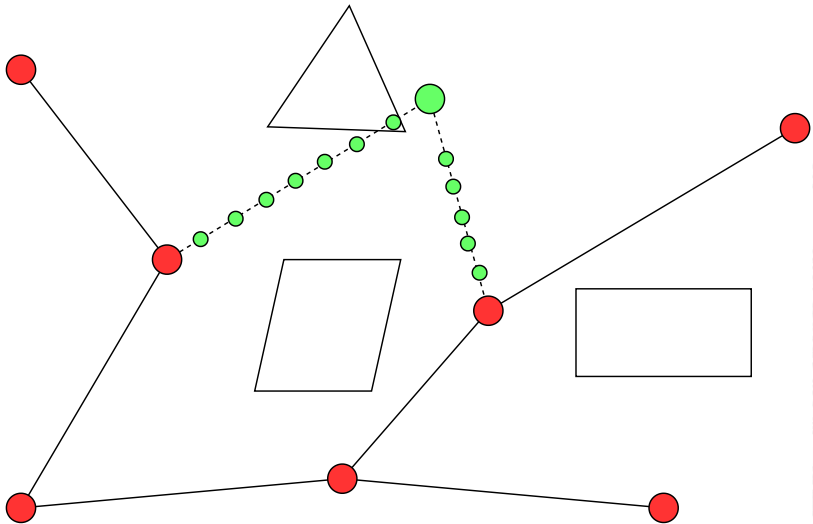
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# Probabilistic Roadmaps

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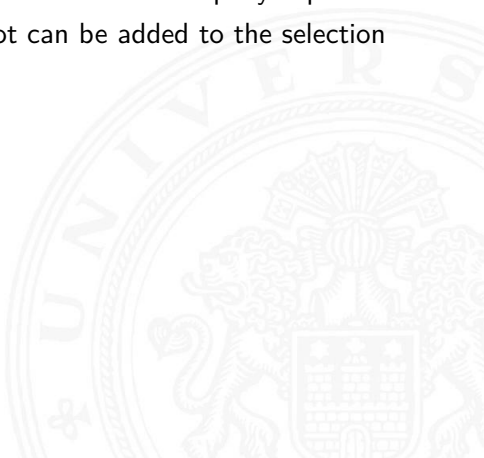
- ▶ single roadmap construction
- ▶ costly for changing environment
- ▶ fast path planning once roadmap has been constructed
- ▶ no guarantee for optimal path





# Rapidly-Exploring Random Trees (RRT)

- ▶ instead of trying to connect a random configuration  $x$  directly to the graph, a configuration  $y$  between the closest and the new random state  $x$  is connected
- ▶ more states are connected while the tree still rapidly expands
- ▶ motion constraints of the robot can be added to the selection function of  $y$





# Rapidly-Exploring Random Trees (RRT)

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- ▶ more states are connected while the tree still rapidly expands
- ▶ motion constraints of the robot can be added to the selection function of  $y$
  
- ▶ live demo <http://demonstrations.wolfram.com/RapidlyExploringRandomTreeRRTAndRRT/>





# Rapidly-Exploring Random Trees (RRT)

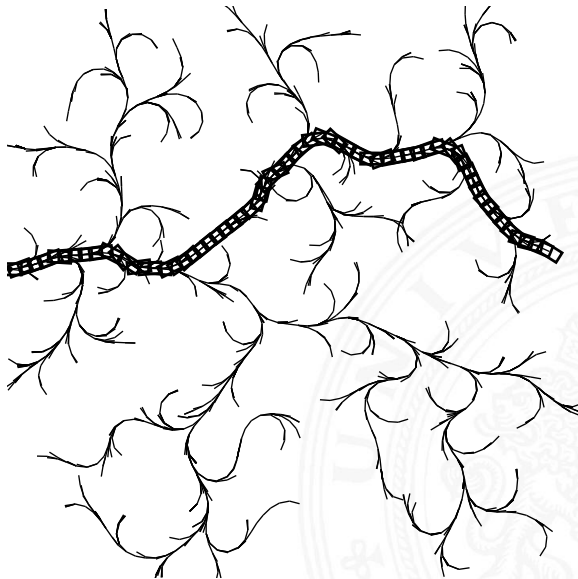
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- ▶ Bidirectional search from start and goal configuration
- ▶ usually outperforms classical RRT algorithm
- ▶ goal and/or start configuration is often cluttered (i.e., close to obstacles for example for grasping)





# Conclusion and Outlook

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- ▶ motion planning is computationally hard but necessary





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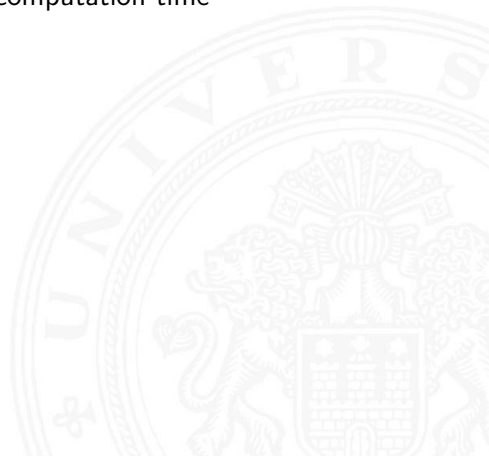
- ▶ motion planning is computationally hard but necessary
- ▶ dynamic environment needs recalculation, therefore we need fast algorithms





# Conclusion and Outlook

- ▶ motion planning is computationally hard but necessary
- ▶ dynamic environment needs recalculation, therefore we need fast algorithms
- ▶ trade-off: optimal solution / computation time





# Conclusion and Outlook

Motivation

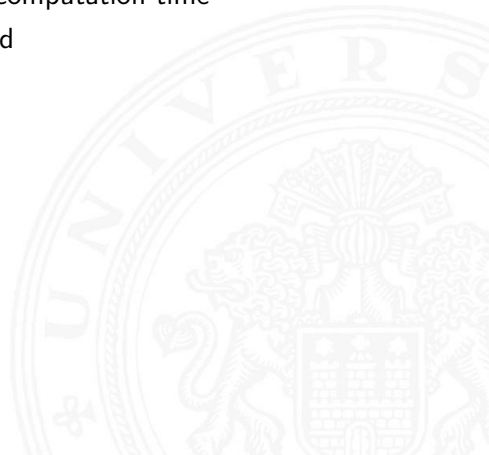
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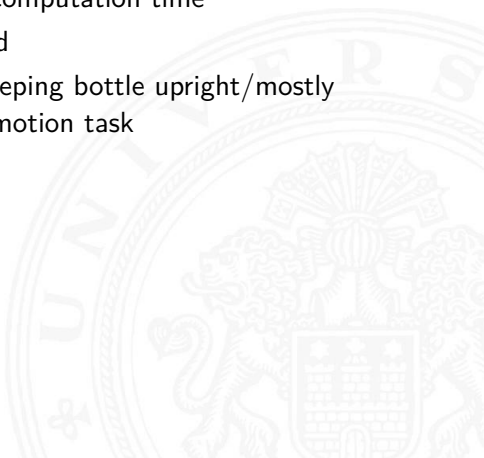
- ▶ motion planning is computationally hard but necessary
- ▶ dynamic environment needs recalculation, therefore we need fast algorithms
- ▶ trade-off: optimal solution / computation time
- ▶ motion prediction not modeled





# Conclusion and Outlook

- ▶ motion planning is computationally hard but necessary
- ▶ dynamic environment needs recalculation, therefore we need fast algorithms
- ▶ trade-off: optimal solution / computation time
- ▶ motion prediction not modeled
- ▶ additional constraints (e.g. keeping bottle upright/mostly upright) may be required for motion task





# References

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# References (cont.)

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