WS 2004/05	Übungen Technische Informatik 1 (T1)	Musterlösung zu Aufgabenblatt 4

Hinweis: Innerhalb der Musterlösung werden zum Teil nur die ODER-Verknüpfungen ($\vee$) explizit angegeben.

Lösung 4.1

a)
$$\begin{aligned} y &= b_0 \wedge (b_0 \vee b_1 \wedge b_2) \vee b_1 \wedge b_2 \wedge b_3 \\ &= b_0[b_0 \vee (b_1 b_2)] \vee b_1 b_2 b_3 \end{aligned}$$

Anwendung des Absorptionsgesetzes:

$$= b_0 \vee b_1 b_2 b_3$$

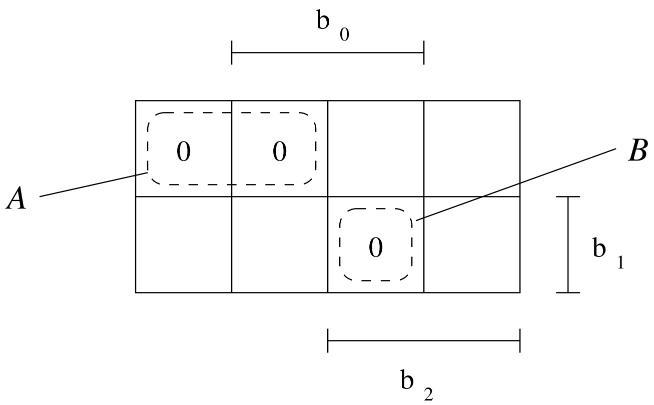
b)
$$\begin{aligned} y &= (b_0 \wedge b_0) \vee (b_0 \wedge b_2) \vee (b_1 \wedge b_2) \vee (b_0 \wedge b_1) \\ &= (b_0 b_0) \vee (b_0 b_2) \vee (b_1 b_2) \vee (b_0 b_1) \end{aligned}$$

Mehrfache Anwendung der Distributivgesetze:

$$\begin{aligned} &= [b_0(b_0 \vee b_2)] \vee [b_1(b_0 \vee b_2)] \\ &= (b_0 \vee b_2)(b_0 \vee b_1) \\ &= b_0 \vee b_1 b_2 \end{aligned}$$

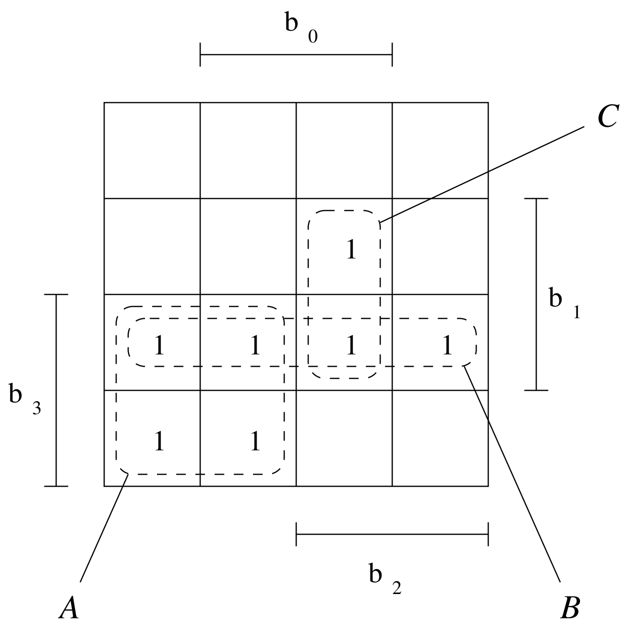
c)
$$\begin{aligned} y &= (b_0 \wedge b_1) \vee (b_0 \wedge b_1) \vee (\overline{b}_0 \wedge b_1) \\ &= b_0 b_1 \vee b_0 b_1 \vee \overline{b}_0 b_1 \\ &= b_0 b_1 \vee \overline{b}_0 b_1 \\ &= b_1 \end{aligned}$$

d)
$$y = (b_0 \vee b_1 \vee b_2) \wedge (\overline{b}_0 \vee b_1 \vee b_2) \wedge (\overline{b}_0 \vee \overline{b}_1 \vee \overline{b}_2)$$



$$= \underbrace{(b_1 \vee b_2)}_{A} \; \underbrace{(\overline{b}_0 \vee \overline{b}_1 \vee \overline{b}_2)}_{B}$$

e) $$\begin{aligned} y &= (b_0 \wedge b_1 \wedge b_2 \wedge \overline{b}_3) \vee (b_0 \wedge b_1 \wedge b_2 \wedge b_3) \vee (\overline{b}_0 \wedge b_1 \wedge b_2 \wedge b_3) \vee \\ &\quad (b_0 \wedge \overline{b}_1 \wedge \overline{b}_2 \wedge b_3) \vee (b_0 \wedge b_1 \wedge \overline{b}_2 \wedge b_3) \vee (\overline{b}_0 \wedge b_1 \wedge \overline{b}_2 \wedge b_3) \vee \\ &\quad (\overline{b}_0 \wedge \overline{b}_1 \wedge \overline{b}_2 \wedge b_3) \\ &= (b_0 b_1 b_2 \overline{b}_3) \vee (b_0 b_1 b_2 b_3) \vee (\overline{b}_0 b_1 b_2 b_3) \vee (b_0 \overline{b}_1 \overline{b}_2 b_3) \vee (b_0 b_1 \overline{b}_2 b_3) \vee (\overline{b}_0 b_1 \overline{b}_2 b_3) \vee \\ &\quad (\overline{b}_0 \overline{b}_1 \overline{b}_2 b_3) \end{aligned}$$



$$= \underbrace{(\overline{b}_2 b_3)}_{A} \vee \underbrace{(b_1 b_3)}_{B} \vee \underbrace{(b_0 b_1 b_2)}_{C}$$

Lösung 4.2

$$f(b_0, b_1, b_2) = (b_0 \vee \overline{b}_1) b_0 \vee (b_0 \vee \overline{b}_1) \overline{b}_2$$

Anwendung des Absorptionsgesetzes:

$$= b_0 \vee (b_0 \vee \overline{b}_1) \overline{b}_2$$

Anwendung des Distributivgesetzes:

$$= b_0 \vee b_0 \overline{b}_2 \vee \overline{b}_1 \overline{b}_2$$

Anwendung des Absorptionsgesetzes:

$$= b_0 \vee \overline{b}_1 \overline{b}_2$$

zweimalige Invertierung:

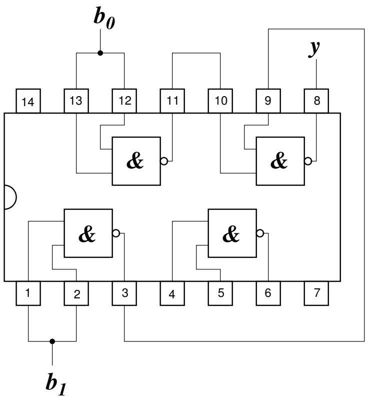
$$= \overline{\overline{b_0 \vee \overline{b}_1 \overline{b}_2}}$$

Anwendung des De Morganschen Gesetzes:

$$= \overline{\overline{b}_0 \wedge \overline{\overline{b}_1 \overline{b}_2}}$$

Lösung 4.3

$$
\begin{aligned}
y &= b_0 \vee b_1 \\
&= \overline{\overline{b_0 \vee b_1}} \\
&= \overline{\overline{b}_0 \wedge \overline{b}_1} \\
&= \overline{\overline{b_0 \wedge b_0} \wedge \overline{b_1 \wedge b_1}}
\end{aligned}
$$



Lösung 4.4

$$
f(b_0, b_1) = b_0 b_1 \vee \overline{b}_0 \overline{b}_1
$$

Umwandlung in KNF durch zweimalige Invertierung und anschließende Anwendung der De Morganschen Gesetze:

$$
\begin{aligned}
&= \overline{\overline{b_0 b_1 \vee \overline{b}_0 \overline{b}_1}} \\
&= \overline{\overline{b_0 b_1} \wedge \overline{\overline{b}_0 \overline{b}_1}} \\
&= \overline{(\overline{b}_0 \vee \overline{b}_1) \wedge (b_0 \vee b_1)}
\end{aligned}
$$

Anwendung des Distributivgesetzes (Ausklammern):

$$
= \overline{\overline{b}_0 b_0 \vee \overline{b}_0 b_1 \vee \overline{b}_1 b_0 \vee \overline{b}_1 b_1}
$$

Eliminierung von redundanten Termen:

$$
= \overline{\overline{b}_0 b_1 \vee b_0 \overline{b}_1}
$$

Anwendung des De Morganschen Gesetzes:

$$
\begin{aligned}
&= \overline{\overline{b}_0 b_1} \wedge \overline{b_0 \overline{b}_1} \\
&= (b_0 \vee \overline{b}_1) \wedge (\overline{b}_0 \vee b_1)
\end{aligned}
$$

Zweimalige Invertierung:

$$
= \overline{\overline{(b_0 \vee \overline{b}_1) \wedge (\overline{b}_0 \vee b_1)}}
$$

Anwendung des De Morganschen Gesetzes:

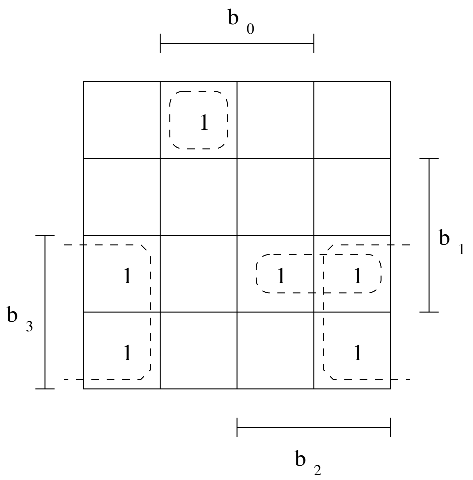
$$
= \overline{\overline{(b_0 \vee \overline{b}_1)} \vee \overline{(\overline{b}_0 \vee b_1)}}
$$

Lösung 4.5

a) Vollständige disjunktive Normalform:

$$f(b_3, b_2, b_1, b_0) = \overline{b}_3\overline{b}_2\overline{b}_1 b_0 \vee b_3\overline{b}_2\overline{b}_1\overline{b}_0 \vee b_3\overline{b}_2 b_1\overline{b}_0 \vee$$
$$b_3 b_2\overline{b}_1\overline{b}_0 \vee b_3 b_2 b_1\overline{b}_0 \vee b_3 b_2 b_1 b_0$$

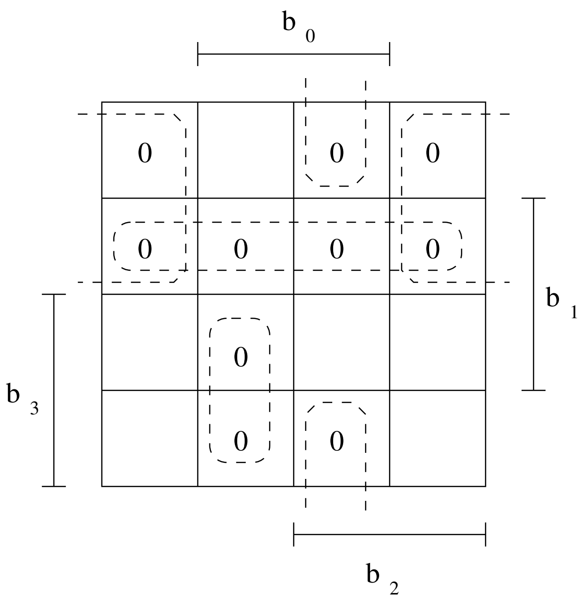
b) Minimierung der vollständigen disjunktiven Normalform über KV-Diagramm:



$$f(b_3, b_2, b_1, b_0) = b_3\overline{b}_0 \vee b_3 b_2 b_1 \vee \overline{b}_3\overline{b}_2\overline{b}_1 b_0$$

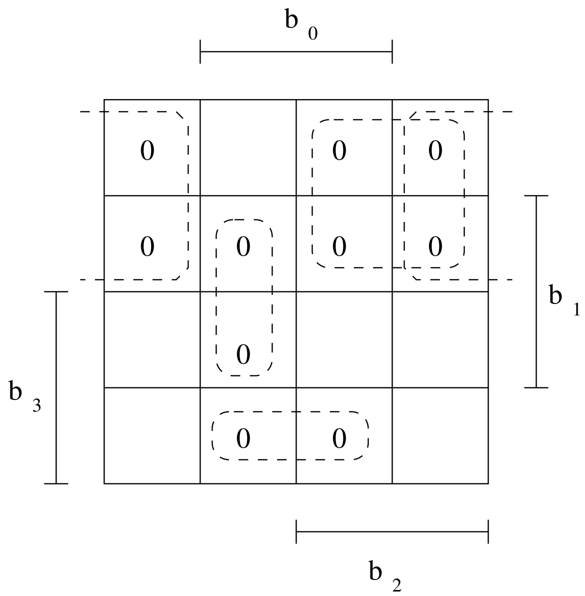
c) Bildung der kanonischen konjunktiven Normalform:

$$f(b_3, b_2, b_1, b_0) = (b_3 \vee b_2 \vee b_1 \vee b_0)(b_3 \vee b_2 \vee \overline{b}_1 \vee b_0)(b_3 \vee b_2 \vee \overline{b}_1 \vee \overline{b}_0)$$
$$(b_3 \vee \overline{b}_2 \vee b_1 \vee b_0)(b_3 \vee \overline{b}_2 \vee b_1 \vee \overline{b}_0)(b_3 \vee \overline{b}_2 \vee \overline{b}_1 \vee b_0)$$
$$(b_3 \vee \overline{b}_2 \vee \overline{b}_1 \vee \overline{b}_0)(\overline{b}_3 \vee b_2 \vee b_1 \vee \overline{b}_0)(\overline{b}_3 \vee b_2 \vee \overline{b}_1 \vee \overline{b}_0)$$
$$(\overline{b}_3 \vee \overline{b}_2 \vee b_1 \vee \overline{b}_0)$$



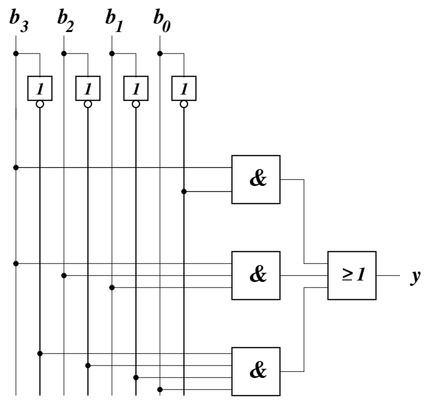
$$f(b_3, b_2, b_1, b_0) = (b_3 \vee \overline{b}_1)(b_3 \vee b_0)(\overline{b}_2 \vee b_1 \vee \overline{b}_0)(\overline{b}_3 \vee b_2 \vee \overline{b}_0)$$

Zweite gleichwertige Lösung:

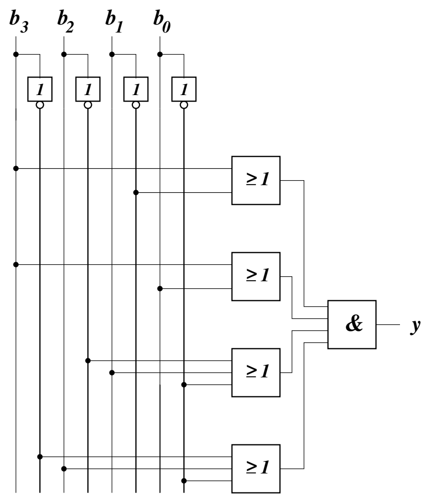


$$f(b_3, b_2, b_1, b_0) \quad = \quad (\overline{b}_2 \vee b_3)(b_0 \vee b_3)(\overline{b}_0 \vee \overline{b}_1 \vee b_2)(\overline{b}_0 \vee b_1 \vee \overline{b}_3)$$

d) Schaltbild zu b):

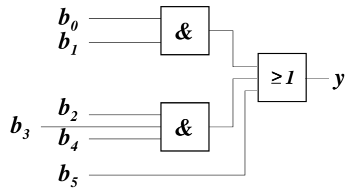


Schaltbild zu c):



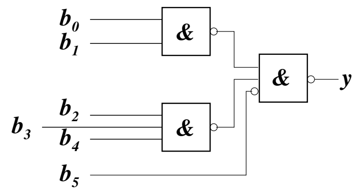
Lösung 4.6

a)

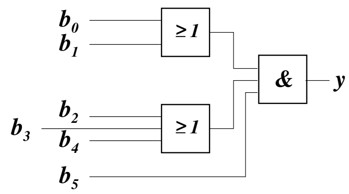


$$\begin{aligned} y &= b_0b_1 \vee b_2b_3b_4 \vee b_5 \\ &= \overline{\overline{b_0b_1 \vee b_2b_3b_4 \vee b_5}} \\ &= \overline{\overline{b_0b_1} \wedge \overline{b_2b_3b_4} \wedge \overline{b_5}} \end{aligned}$$

Resultierendes NAND-Netz:

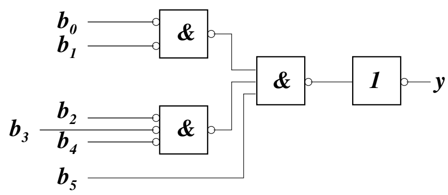


b)

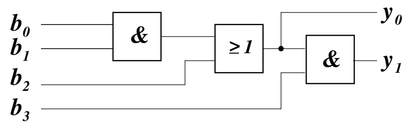


$$\begin{aligned} y &= (b_0 \vee b_1) \wedge (b_2 \vee b_3 \vee b_4) \wedge b_5 \\ &= \overline{\overline{(b_0 \vee b_1)}} \wedge \overline{\overline{(b_2 \vee b_3 \vee b_4)}} \wedge b_5 \\ &= \overline{\overline{b}_0 \wedge \overline{b}_1} \wedge \overline{\overline{b}_2 \wedge \overline{b}_3 \wedge \overline{b}_4} \wedge b_5 \\ &= \overline{\overline{\overline{\overline{b}_0 \wedge \overline{b}_1} \wedge \overline{\overline{b}_2 \wedge \overline{b}_3 \wedge \overline{b}_4} \wedge b_5}} \end{aligned}$$

Resultierendes NAND-Netz:

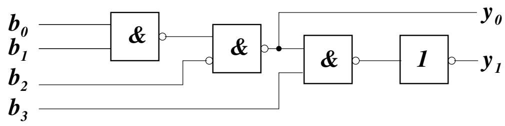


c)



$$
\begin{aligned}
y_0 &= b_0b_1 \lor b_2 \\
&= \overline{\overline{b_0b_1 \lor b_2}} \\
&= \overline{\overline{b_0b_1} \land \overline{b_2}} \\
y_1 &= y_0b_3 \\
&= \overline{\overline{y_0b_3}}
\end{aligned}
$$

Resultierendes NAND-Netz:



Lösung 4.7

a)

$$
\begin{aligned}
y &= \overline{\overline{\overline{(\overline{b_1\overline{b}_2b_3} \land \overline{\overline{b}_1b_2})} \land b_0} \land \overline{\overline{(b_2 \oplus b_3)} \land \overline{b}_0}} \\
&= \overline{\overline{\overline{(\overline{b_1\overline{b}_2b_3} \land \overline{\overline{b}_1b_2})} \land b_0}} \lor \overline{\overline{\overline{(b_2 \oplus b_3)} \land \overline{b}_0}} \\
&= \overline{(\overline{b_1\overline{b}_2b_3} \land \overline{\overline{b}_1b_2})} \land b_0 \lor \overline{(b_2 \oplus b_3)} \land \overline{b}_0 \\
&= \overline{\overline{(b_1\overline{b}_2b_3 \lor \overline{b}_1b_2)}} \land b_0 \lor (b_2b_3 \lor \overline{b}_2\overline{b}_3) \land \overline{b}_0 \\
&= b_0b_1\overline{b}_2b_3 \lor b_0\overline{b}_1b_2 \lor \overline{b}_0b_2b_3 \lor \overline{b}_0\overline{b}_2\overline{b}_3
\end{aligned}
$$

b)